```
In[ ]:= Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

```
First we import some formatting...

...okay, that's better, from now on any commentary written inside this
  Calibration.m wrapper will present as blue text (i.e. this text is not
  part of PSALTer, it is just a use-case). Next we load the PSALTer package:

------------------------------------------------------------

Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}

CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.

Connecting to external linux executable...

Connection established.

------------------------------------------------------------

Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}

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------------------------------------------------------------

Package xAct`xPert`  version 1.0.6, {2018, 2, 28}

CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia
  and Guillermo A. Mena Marugan, under the General Public License.

** Variable $PrePrint assigned value ScreenDollarIndices

** Variable $CovDFormat changed from Prefix to Postfix

** Option AllowUpperDerivatives of ContractMetric changed from False to True

** Option MetricOn of MakeRule changed from None to All

** Option ContractMetrics of MakeRule changed from False to True

------------------------------------------------------------

Package xAct`Invar`  version 2.0.5, {2013, 7, 1}

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  D. Yllanes and R. Portugal, under the General Public License.

** DefConstantSymbol: Defining constant symbol sigma.

** DefConstantSymbol: Defining constant symbol dim.

** Option CurvatureRelations of DefCovD changed from True to False

** Variable $CommuteCovDsOnScalars changed from True to False

------------------------------------------------------------

Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}

CopyRight (C) 2005-2021, David Yllanes and
  Jose M. Martin-Garcia, under the General Public License.
```

```
——————————————————————————————————————————————————————————————
Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}
CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.
——————————————————————————————————————————————————————————————
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
** Variable $CovDFormat changed from Postfix to Prefix
** Option CurvatureRelations of DefCovD changed from False to True
——————————————————————————————————————————————————————————————
Package xAct`HiGGS`  version 2.0.0-developer, {2023, 1, 13}
CopyRight © 2022, Will E. V. Barker and Manuel Hohmann, under the General Public License.
——————————————————————————————————————————————————————————————
HiGGS incorporates code by Cyril Pitrou.
——————————————————————————————————————————————————————————————
Package xAct`PSALTer` version 1.0.0-developer, {2023, 1, 13}
CopyRight © 2022, Will E. V. Barker and C. Rew, under the General Public License.
——————————————————————————————————————————————————————————————
These packages come with ABSOLUTELY NO WARRANTY; for details type
  Disclaimer[]. This is free software, and you are welcome to redistribute
  it under certain conditions. See the General Public License for details.
——————————————————————————————————————————————————————————————
```

Now we set up the general Lagrangian:

$$-\lambda\, \mathcal{R}^{ij}{}_{ij} + \left(\frac{r_1}{3}+\frac{r_2}{6}\right)\mathcal{R}_{ij\hbar l}\,\mathcal{R}^{ij\hbar l} + \left(\frac{2 r_1}{3}-\frac{2 r_2}{3}\right)\mathcal{R}_{i\hbar jl}\,\mathcal{R}^{ij\hbar l} +$$

$$(r_4+r_5)\,\mathcal{R}_{i}{}^{l}{}_{jl}\,\mathcal{R}^{i\hbar j}{}_{\hbar} + (r_4-r_5)\,\mathcal{R}^{i\hbar j}{}_{\hbar}\,\mathcal{R}_{j}{}^{l}{}_{il} + \left(\frac{r_1}{3}+\frac{r_2}{6}-r_3\right)\mathcal{R}^{ij\hbar l}\,\mathcal{R}_{\hbar lij} +$$

$$\left(\frac{\lambda}{4}+\frac{t_1}{3}+\frac{t_2}{12}\right)\mathcal{T}_{ij\hbar}\,\mathcal{T}^{ij\hbar} + \left(-\frac{\lambda}{2}-\frac{t_1}{3}+\frac{t_2}{6}\right)\mathcal{T}^{ij\hbar}\,\mathcal{T}_{j\hbar i} + \left(-\lambda-\frac{t_1}{3}+\frac{2 t_3}{3}\right)\mathcal{T}_{i}{}^{ji}\,\mathcal{T}_{\hbar j}{}^{\hbar}$$

We also knock up some simple tools to linearise the Lagrangian:

```
** DefConstantSymbol: Defining constant symbol PerturbativeParameter.
```

Now we would like to check the basic Einstein-Cartan theory. Here is the full nonlinear Lagrangian:

$$t_1\, \mathcal{R}^{ij}{}_{ij}$$

To use PSALTer, you have to first linearise this Lagrangian to second order around the desired vacuum:

$$\underset{1}{t}\,\mathcal{A}_{\alpha j i}\,\mathcal{A}^{\alpha i j}+\underset{1}{t}\,\mathcal{A}^{\alpha i}{}_{\alpha}\,\mathcal{A}_{i}{}^{j}{}_{j}+2\,\underset{1}{t}\,f^{\alpha i}\,\partial_{i}\mathcal{A}_{\alpha}{}^{j}{}_{j}-2\,\underset{1}{t}\,\partial_{i}\mathcal{A}^{\alpha i}{}_{\alpha}-2\,\underset{1}{t}\,f^{\alpha i}\,\partial_{j}\mathcal{A}_{\alpha}{}^{j}{}_{i}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) a-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}0 & \frac{i k \underset{1}{t}}{\sqrt{2}} & 0\\ -\frac{i k \underset{1}{t}}{\sqrt{2}} & -\underset{1}{t} & -i\sqrt{\frac{3}{2}}\,k\,\underset{1}{t}\\ 0 & i\sqrt{\frac{3}{2}}\,k\,\underset{1}{t} & 0\end{pmatrix},\ \left(-\underset{1}{t}\right),\ \begin{pmatrix}0 & \frac{i k \underset{1}{t}}{\sqrt{2}} & 0\\ -\frac{i k \underset{1}{t}}{\sqrt{2}} & -\frac{\underset{1}{t}}{2} & -\frac{\underset{1}{t}}{\sqrt{2}}\\ 0 & -\frac{\underset{1}{t}}{\sqrt{2}} & 0\end{pmatrix},\ \begin{pmatrix}0 & 0 & 0 & 0\\ 0 & -\frac{\underset{1}{t}}{2} & i k \underset{1}{t} & \frac{\underset{1}{t}}{\sqrt{2}}\\ 0 & -i k \underset{1}{t} & 0 & 0\\ 0 & \frac{\underset{1}{t}}{\sqrt{2}} & 0 & 0\end{pmatrix},\ \begin{pmatrix}0 & \frac{i k \underset{1}{t}}{\sqrt{2}}\\ -\frac{i k \underset{1}{t}}{\sqrt{2}} & \frac{\underset{1}{t}}{2}\end{pmatrix},\ \left(\frac{\underset{1}{t}}{2}\right)\right\}$$

Gauge constraints on source currents:

$$\left\{\underset{\cdot}{0^{+}}\tau^{\flat\parallel}==\underset{\cdot}{0^{+}}\tau^{\flat\perp},\ i\,\underset{\cdot}{1^{+}}\tau^{\flat\parallel\,\alpha b}==k\,\underset{\cdot}{1^{+}}\sigma^{\flat\perp\,\alpha b},\ i\,\underset{\cdot}{1^{-}}\tau^{\flat\perp\,\alpha}==2\,k\,\underset{\cdot}{1^{-}}\sigma^{\flat\perp\,\alpha},\ \underset{\cdot}{1^{-}}\tau^{\flat\parallel\,\alpha}==0\right\}$$

The Drazin (Moore-Penrose) inverses of these a-matrices, which are functionally analogous to the inverse b-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix}\frac{1}{8k^{2}\underset{1}{t}} & \frac{i}{2\sqrt{2}\,k\underset{1}{t}} & \frac{\sqrt{3}}{8k^{2}\underset{1}{t}}\\ -\frac{i}{2\sqrt{2}\,k\underset{1}{t}} & 0 & -\frac{i\sqrt{\frac{3}{2}}}{2k\underset{1}{t}}\\ \frac{\sqrt{3}}{8k^{2}\underset{1}{t}} & \frac{i\sqrt{\frac{3}{2}}}{2k\underset{1}{t}} & \frac{3}{8k^{2}\underset{1}{t}}\end{pmatrix},\ \left(-\frac{1}{\underset{1}{t}}\right),\ \begin{pmatrix}\frac{k^{2}}{(1+k^{2})^{2}\underset{1}{t}} & \frac{i\sqrt{2}\,k}{\underset{1}{t}+k^{2}\underset{1}{t}} & -\frac{i k}{(1+k^{2})^{2}\underset{1}{t}}\\ -\frac{i\sqrt{2}\,k}{\underset{1}{t}+k^{2}\underset{1}{t}} & 0 & -\frac{\sqrt{2}}{\underset{1}{t}+k^{2}\underset{1}{t}}\\ \frac{i k}{(1+k^{2})^{2}\underset{1}{t}} & -\frac{\sqrt{2}}{\underset{1}{t}+k^{2}\underset{1}{t}} & \frac{1}{(1+k^{2})^{2}\underset{1}{t}}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}0 & 0 & 0 & 0\\ 0 & 0 & \frac{2ik}{\underset{1}{t}+2k^{2}\underset{1}{t}} & \frac{\sqrt{2}}{\underset{1}{t}+2k^{2}\underset{1}{t}}\\ 0 & -\frac{2ik}{\underset{1}{t}+2k^{2}\underset{1}{t}} & \frac{2k^{2}}{(1+2k^{2})^{2}\underset{1}{t}} & -\frac{i\sqrt{2}\,k}{(1+2k^{2})^{2}\underset{1}{t}}\\ 0 & \frac{\sqrt{2}}{\underset{1}{t}+2k^{2}\underset{1}{t}} & \frac{i\sqrt{2}\,k}{(1+2k^{2})^{2}\underset{1}{t}} & \frac{1}{(1+2k^{2})^{2}\underset{1}{t}}\end{pmatrix},\ \begin{pmatrix}-\frac{1}{k^{2}\underset{1}{t}} & \frac{i\sqrt{2}}{k\underset{1}{t}}\\ -\frac{i\sqrt{2}}{k\underset{1}{t}} & 0\end{pmatrix},\ \left(\frac{2}{\underset{1}{t}}\right)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{-\frac{9p^{2}}{\underset{1}{t}},\ -\frac{9p^{2}}{\underset{1}{t}}\right\}$$

Overall unitarity conditions:

$$\left(p < 0 \,\&\&\, t_{\dot{1}} < 0\right) \,||\, \left(p > 0 \,\&\&\, t_{\dot{1}} < 0\right)$$

Okay, so that is the end of the PSALTer output for Einstein-Cartan gravity. What we find are no propagating massive modes, but instead two degrees of freedom in the massive sector. The no-ghost conditions on these massless d.o.f restrict the sign in front of the Einstein-Hilbert term to be negative (which is what we expect for our conventions).

Using Karananas' coefficients, it is particularly easy to also look at GR, instead of Einstein-Cartan theory. The difference here is that the quadratic torsion coefficients are manually removed. Here is the nonlinear Lagrangian:

$$-\lambda_{\dot{}}\, \mathcal{R}^{ij}{}_{ij} + \frac{1}{4}\lambda_{\dot{}}\, \mathcal{T}_{ijk}\, \mathcal{T}^{ijk} + \frac{1}{2}\lambda_{\dot{}}\, \mathcal{T}^{ijk}\, \mathcal{T}_{jik} + \lambda_{\dot{}}\, \mathcal{T}^{i}{}_{i}{}^{j}\, \mathcal{T}^{k}{}_{jk}$$

Here is the linearised theory:

$$-2\lambda_{\dot{}}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial_{a} f^{aa'} - 2\lambda_{\dot{}}\, f^{aa'}\, \partial_{a'}\mathcal{A}_{a}{}^{i}{}_{i} + 2\lambda_{\dot{}}\, \partial_{a'}\mathcal{A}^{aa'}{}_{a} + 2\lambda_{\dot{}}\, \mathcal{A}_{a'}{}^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} - \lambda_{\dot{}}\, \partial_{a'} f^{i}{}_{i}\, \partial^{a'} f^{a}{}_{a} + 2\lambda_{\dot{}}\, f^{aa'}\, \partial_{i}\mathcal{A}_{a}{}^{i}{}_{a'} - \lambda_{\dot{}}\, \partial_{a} f^{aa'}\, \partial_{i} f^{i}{}_{a'} + 2\lambda_{\dot{}}\, \partial^{a'} f^{a}{}_{a}\, \partial_{i} f^{i}{}_{a'} + 2\lambda_{\dot{}}\, \mathcal{A}_{a'ia}\, \partial^{i} f^{aa'} - \lambda_{\dot{}}\, \partial_{a} f_{a'i}\, \partial^{i} f^{aa'} + \frac{1}{2}\lambda_{\dot{}}\, \partial_{a} f_{ia'}\, \partial^{i} f^{aa'} - \frac{1}{2}\lambda_{\dot{}}\, \partial_{a'} f_{ai}\, \partial^{i} f^{aa'} + \frac{1}{2}\lambda_{\dot{}}\, \partial_{i} f_{aa'}\, \partial^{i} f^{aa'} + \frac{1}{2}\lambda_{\dot{}}\, \partial_{i} f_{a'a}\, \partial^{i} f^{aa'}$$

Now we pass this theory into the PSALTer package, which computes the particle spectrum:

The (possibly singular) *a*-matrices associated with the Lagrangian, as defined below Eq. (18) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -2k^2\lambda_{\dot{}} & -\frac{3ik\lambda_{\dot{}}}{\sqrt{2}} & 0 \\ \frac{3ik\lambda_{\dot{}}}{\sqrt{2}} & 0 & i\sqrt{\frac{3}{2}}\,k\lambda_{\dot{}} \\ 0 & -i\sqrt{\frac{3}{2}}\,k\lambda_{\dot{}} & 0 \end{pmatrix}, (0), \begin{pmatrix} 0 & -i\sqrt{2}\,k\lambda_{\dot{}} & 0 \\ i\sqrt{2}\,k\lambda_{\dot{}} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -ik\lambda_{\dot{}} & 0 & 0 \\ ik\lambda_{\dot{}} & 0 & -ik\lambda_{\dot{}} & 0 \\ 0 & ik\lambda_{\dot{}} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} k^2\lambda_{\dot{}} & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Gauge constraints on source currents:

$$\left\{ {}^{0^-}_{\cdot}\sigma^{\flat\parallel} == 0,\ {}^{1^+}_{\cdot}\sigma^{\flat\perp}{}^{ab} == 0,\ {}^{1^-}_{\cdot}\sigma^{\flat\perp}{}^{a} == 0,\ {}^{1^-}_{\cdot}\tau^{\flat\parallel}{}^{a} + {}^{1^-}_{\cdot}\tau^{\flat\perp}{}^{a} == 0,\ {}^{2^+}_{\cdot}\sigma^{\flat\parallel}{}^{ab} == 0,\ {}^{2^-}_{\cdot}\sigma^{\flat\parallel}{}^{abc} == 0 \right\}$$

The Drazin (Moore-Penrose) inverses of these *a*-matrices, which are functionally analogous to the inverse *b*-matrices described below Eq. (21) of arXiv:1812.02675:

$$\left\{\begin{pmatrix} -\frac{1}{2k^2\lambda.} & 0 & \frac{\sqrt{3}}{2k^2\lambda.} \\ 0 & 0 & \frac{i\sqrt{\frac{2}{3}}}{k\lambda.} \\ \frac{\sqrt{3}}{2k^2\lambda.} & -\frac{i\sqrt{\frac{2}{3}}}{k\lambda.} & -\frac{3}{2k^2\lambda.} \end{pmatrix}, (0), \begin{pmatrix} 0 & -\frac{i}{\sqrt{2}\,k\lambda.} & 0 \\ \frac{i}{\sqrt{2}\,k\lambda.} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -\frac{i}{2k\lambda.} & 0 & 0 \\ \frac{i}{2k\lambda.} & 0 & -\frac{i}{2k\lambda.} & 0 \\ 0 & \frac{i}{2k\lambda.} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} \frac{1}{k^2\lambda.} & 0 \\ 0 & 0 \end{pmatrix}, (0)\right\}$$

Square masses:

{{}, {}, {}, {}, {}, {}}

Massive pole residues:

{{}, {}, {}, {}, {}, {}}

Massless eigenvalues:

$$\left\{\frac{p^2}{\lambda.}, \frac{p^2}{\lambda.}\right\}$$

Overall unitarity conditions:

$$(p < 0 \;\&\&\; \lambda. > 0) \,||\, (p > 0 \;\&\&\; \lambda. > 0)$$

Thus, the conclusions are the same, as expected.