Dynamical vectors but no strong coupling near the geometrical trinity of gravity

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We identify points near the geometric trinity of gravity, positioned between general relativity, its teleparallel equivalent, and Einstein–Cartan theory, in which the conventional multipliers suppress only part of the torsion. In this configuration, a quadratic curvature invariant can be added without introducing strongly-coupled torsion modes. The total effect is to augment GR with either a vector or pseudovector Proca field, which couples to the matter spin current(s).

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Introduction. – Gravity may be coupled to fermions by gauging the Lorentz group [1–3], and so augmenting the Levi–Civita connection $\mathring{\Gamma}^{\mu}_{\lambda\nu}$ with spacetime torsion $\Gamma^{\mu}_{\lambda\nu} \equiv \mathring{\Gamma}^{\mu}_{\lambda\nu} - \frac{1}{2}\big(T^{\mu}{}_{\lambda\nu} - T_{n}{}^{\mu}{}_{\lambda} + T_{\lambda\nu}{}^{\mu}\big)$. The conservative Einstein–Cartan (EC) approach to including torsion [1, 4, 5] retains the Einstein–Hilbert action $L_{\rm EC} \equiv -\frac{1}{2}{M_{\rm Pl}}^2 R$, but uses the *Riemann–Cartan* curvature scalar[1] R. In the presence of matter, the torsion integrates out algebraically [1, 6, 7] to give the effective theory in $g_{\mu\nu}$ of general relativity (GR), but where the symmetrised matter stress-energy tensor now receives $\sim \bar{\psi}\psi\bar{\psi}\psi/{M_{\rm Pl}}^2$ fermionic corrections [8, 9]. Plausibly, such corrections may only become relevant in the very early Universe [9–12] or within black holes [10, 13–15], where they may remove singularities. EC theory reverts to GR when the torsion is suppressed by a multiplier field

$$L_{\rm GR} \equiv -\frac{1}{2}{M_{\rm Pl}}^2 \mathring{R} \simeq -\frac{1}{2}{M_{\rm Pl}}^2 R + \lambda_{\mu\nu\sigma}T^{\mu\nu\sigma}. \tag{1}$$

It is a remarkable fact that linearly connected spacetime encompasses *two* alterntive realisations of free GR. The 'metric' teleparallel equivalent of GR (TEGR) [16]

$$\begin{aligned} L_{\rm TEGR} \equiv & \frac{4}{9}{M_{\rm Pl}}^2 {}^{(1)}T_{\mu[\nu\sigma]}\, {}^{(1)}T^{\mu[\nu\sigma]} - \frac{1}{3}{M_{\rm Pl}}^2 {}^{(2)}T_{\mu}\, {}^{(2)}T^{\mu} \\ & + \frac{3}{4}{M_{\rm Pl}}^2 {}^{(3)}T_{\mu}\, {}^{(3)}T^{\mu} + \lambda^{\mu\nu}{}_{\sigma\lambda}R_{\mu\nu}{}^{\sigma\lambda}, \end{aligned} \tag{2}$$

— where ${}^{(1)}T^{\mu\nu\sigma}$, ${}^{(2)}T^{\mu}$ and ${}^{(3)}T^{\mu}$ are the tensor, vector and pseudovector irreducible parts of the torsion[2] — has the property $L_{\rm GR} \simeq L_{\rm TEGR}$ despite suppressing the Riemann curvature with a multiplier field. The 'symmetric' alternative (STEGR) [17] is reached by relaxing the metricity condition $\nabla_{\mu}g_{\nu\sigma} \equiv 0$ while suppressing both curvature and torsion. Normal GR as in (1) implicitly enforces metricity via yet another multiplier. The familiar IR limit of free gravity is recovered at each vertex in this *geometrical trinity* [18, 19].

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[1] We work with the 'West Coast' signature $(+,-,-,-)$ and use the conventions $R \equiv R^{\lambda}{}_{\lambda}$ and $R_{\mu\nu} \equiv R^{\lambda}{}_{\mu\lambda\nu}$ for $R_{\mu\nu\lambda}{}^{\sigma} \equiv 2\partial_{[\nu}\Gamma^{\sigma}_{\mu]\lambda} + 2\Gamma^{\sigma}_{[\nu|\kappa}\Gamma^{\kappa}_{|\mu]\lambda}$, and analogously for the Riemann tensor $\mathring{R}_{\mu\nu\lambda}{}^{\sigma}$ in terms of $\mathring{\Gamma}^{\sigma}_{\mu\nu}$.

[2] We use the decomposition $T^{\mu}{}_{\nu\sigma} \equiv \frac{4}{3}{}^{(1)}T^{\mu}{}_{[\nu\sigma]} + \frac{2}{3}\delta^{\mu}_{[\nu}\, {}^{(2)}T_{\sigma]} + \epsilon^{\mu}{}_{\nu\sigma\lambda}\, {}^{(3)}T^{\lambda}$.

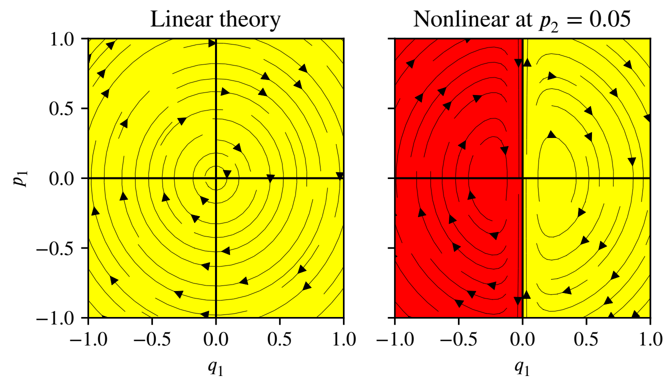

FIG. 1. Strong coupling in the system $L = \dot{q}_1^2 - q_1^2 + q_1^2\dot{q}_2^2 - q_2^2$. The perturbative 'approximation' is $L \approx \dot{q}_1^2 - q_1^2 - q_2^2$, but this predicts an erroneous vacuum $q_2 = q_1 = p_1 = 0$ (left), where the true double oscillator (right) has a fatal separatrix. Strongly-coupled d.o.f, analogous to q_2, reveal themselves via structural changes to the Hamiltonian constraint algebra when passing from linearised to nonlinear gravity theories. In such cases it is not clear how background of the linearisation can then be a viable spacetime, even though it may solve the nonlinear field equations.

As illustrated in Fig. 1, attempts to explore the landscape surrounding the trinity vertices – be they Wilsonian extensions out of the IR, or modified gravity theories – are crippled by the *strong coupling problem* [20–25]. Higher-order operators typically introduce degrees of freedom (d.o.f) which are not captured by the linearised particle spectra [22–25]. Strongly coupled torsion modes are also guaranteed to be ghosts by the same conditions which keep the linear theory unitary [26–30]. Regardless of the implications for the S-matrix, it is understood that strongly coupled modes render the background (Minkowski) spacetime dynamically unreachable [31].

This letter presents a 'dynamically consistent' (DC) theory

$$\begin{aligned} L_{\rm DC} \equiv & -\frac{1}{2}{M_{\rm Pl}}^2 R + \hat{\alpha}_5 R_{[\mu\nu]}R^{[\mu\nu]} + {}^{(1)}\lambda_{\mu\nu\sigma}\, {}^{(1)}T^{\mu\nu\sigma} \\ & + \hat{\beta}_2 {M_{\rm Pl}}^2 {}^{(2)}T_{\mu}\, {}^{(2)}T^{\mu} + \hat{\beta}_3 {M_{\rm Pl}}^2 {}^{(3)}T_{\mu}\, {}^{(3)}T^{\mu}, \end{aligned} \tag{3}$$

which is reached (e.g. by the Wilsonian expansion) from a point *between* GR in (1) and TEGR in (2). In isolation, the invariant $R_{[\mu\nu]}R^{[\mu\nu]}$ — forbidden in GR due to $\mathring{R}_{[\mu\nu]} \equiv 0$ — is also a forbidden higher-order correction to EC gravity because it introduces six strongly coupled d.o.f [20, 27, 29]. Indeed, it is commonly thought that only *scalar* torsion can safely propagate in a $L \sim R + R^2 + T^2$ theory [30, 32–35]. However we show that this outlook only holds when the usual multipliers act to disable torsion or curvature in their entirety. By splitting multipliers between individual $\mathrm{SO}^+(1,3)$ torsion irreps, the DC theory allows, for the first time, some higher-spin *vector* torsion to propagate in a dynamically consistent manner.

This mechanism opens up the route to finding viable archipelagos around the geometric trinity. The more prosaic objection still remains: the affected modes tend to be ghosts anyway [26–30], so removing them entirely seems preferable to rendering them perturbative. Indeed, we will find that in (3) the ${}^{(2)}T_\mu$ and ${}^{(3)}T_\mu$ are Proca fields of bare mass[3]

$$ {}^{(2)}m^2 = -\frac{3{M_{\mathrm{Pl}}}^2(1+2\hat{\beta}_2)}{4\hat{\alpha}_5}, \tag{4a} $$

$$ {}^{(3)}m^2 = -\frac{3{M_{\mathrm{Pl}}}^2(1+8\hat{\beta}_3)}{4\hat{\alpha}_5}, \tag{4b} $$

but also that these are ghosts unless $\hat{\alpha}_5 > 0$ and $\hat{\alpha}_5 < 0$ respectively. The solution, whatever the sign of $\hat{\alpha}_5$, is the simple modification $L_{\mathrm{DC}} + {}^{(2)}\lambda_\mu {}^{(2)}T^\mu$ or $L_{\mathrm{DC}} + {}^{(3)}\lambda_\mu {}^{(3)}T^\mu$. Either ghost is then suppressed, whilst still evading strong coupling of the remaining, unitary Proca field.

Nonlinear Hamiltonian analysis. – The mechanism by which strong coupling is eliminated should be demonstrated in the Hamiltonian framework [37–41], since this is how the problem is usually identified [20, 32, 42]: we now do this for the single-Proca case $L_{\mathrm{DC}} + {}^{(2)}\lambda_\mu {}^{(2)}T^\mu$, while referring to Fig. 2.

In non-Riemannian gravity the gauge fields may be taken as the tetrad $e^i{}_\mu$, where $g_{\mu\nu} \equiv \eta_{ij} e^i{}_\mu e^j{}_\nu$, and independent spin connection $\omega^{ij}{}_\mu \equiv \omega^{[ij]}{}_\mu$ [38], and we must also count with these the multipliers ${}^{(1)}\lambda_{\mu\nu}{}^\sigma$ and ${}^{(2)}\lambda_\mu$ [43]. As is typical of gauge theories such as (3), many (spin-parity, J^P) parts of the X-field momenta $\pi_X^{J^P} \equiv \partial\mathcal{L}/\partial\dot{X}^{J^P}$, for $\mathcal{L} = e\big(L_{\mathrm{DC}} + {}^{(2)}\lambda_\mu {}^{(2)}T^\mu\big)$ and $e \equiv \det e^i{}_\mu$, cannot be inverted for the velocities $\dot{X}^{J^P}$ [37, 38, 41]. Instead, their definitions imply *primary* constraints $\phi_X^{J^P} \approx 0$ which vanish 'on-shell' [29, 30, 36]. For example the $\phi_{{}^{(1)}\lambda}^{J^P}$ and $\phi_{{}^{(2)}\lambda}^{J^P}$ are just defined as all the momenta of ${}^{(1)}\lambda_{\mu\nu}{}^\sigma$ and ${}^{(2)}\lambda_\mu$. For a complete Hamiltonian description of the dynamics, *Hamiltonian multipliers* $u_X^{J^P}$ must be introduced to the phase space, emulating missing $\dot{X}^{J^P}$. If $\mathcal{H}$ is the Legendre-transformed $\mathcal{L}$, the Hamiltonian $\mathcal{H}' \equiv \mathcal{H} + \sum_{X,J,P} \phi_X^{J^P} u_X^{J^P}$ can describe the complete dynamics for all solutions to the $u_X^{J^P}$ which maintain $\phi_X^{J^P} \approx 0$.

In the linear theory near Minkowski spacetime, many of the $u_X^{J^P}$ can be determined at once by considering that the the $\phi_X^{J^P}$

[3] We avoid extra symmetries by assuming $\hat{\beta}_2 \neq -1/2$ and $\hat{\beta}_3 \neq -1/8$ [36].

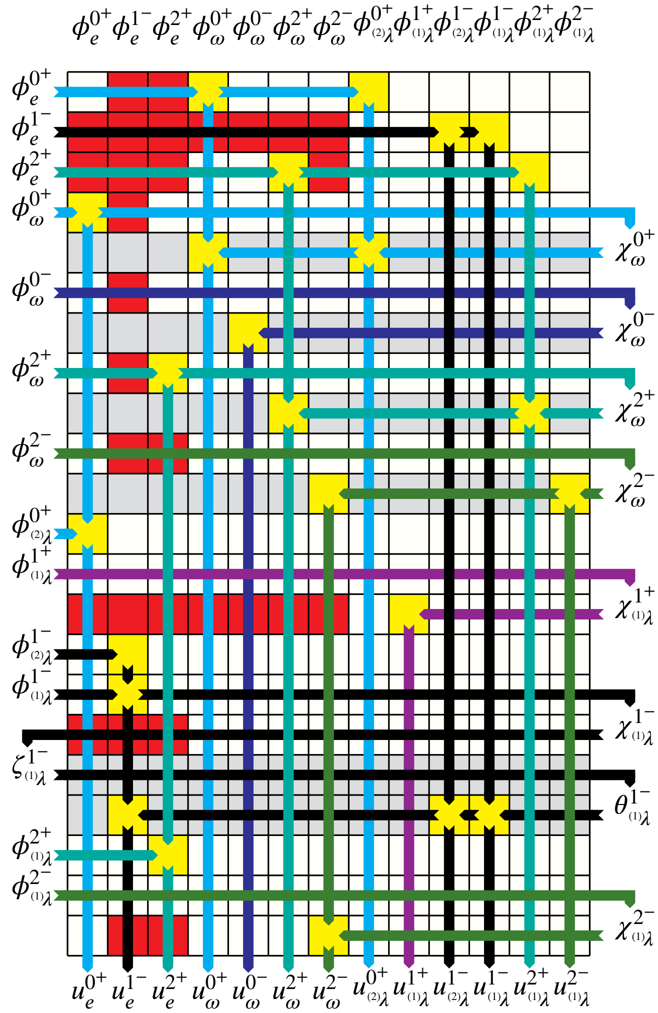

FIG. 2. The constraint algebra of $L_{\mathrm{DC}} + {}^{(2)}\lambda_\mu {}^{(2)}T^\mu$ as defined from (3), as it appears in its simplest form on the constraint shell. All Hamiltonian multipliers $u_X^{J^P}$ are eventually determined by satisfying the consistency conditions (5) of the primary constraints $\phi_X^{J^P}$ (lines coloured by J^P) via abundant $\mathcal{O}(1)$ Poisson brackets (yellow squares). Perturbative brackets (red squares) do not reduce the number of induced constraints $\chi_X^{J^P}$, $\zeta_X^{J^P}$ and $\theta_X^{J^P}$, which would otherwise indicate strong coupling. Some brackets (grey squares) were not computed in detail for this work. Subtracting the $\sum_J 2J+1$ constrained multiplicities leaves the three extra d.o.f of a Proca theory.

must be static to remain zero, i.e. they must have vanishing Poisson brackets $\{\phi_X^{J^P}, \mathcal{H}'\} \approx 0$, or

$$ \{\phi_X^{J^P}, \mathcal{H}\} + \sum_{X',J',P'} \{\phi_X^{J^P}, \phi_{X'}^{J'^{P'}}\} u_{X'}^{J'^{P'}} \approx 0. \tag{5} $$

In particular, the $\phi_{{}^{(2)}\lambda}^{J^P}$ and some $\phi_{{}^{(1)}\lambda}^{J^P}$ have $\mathcal{O}(1)$ brackets with the $\phi_e^{J^P}$ which determine via (5) the $u_{{}^{(2)}\lambda}^{J^P}$, some $u_{{}^{(1)}\lambda}^{J^P}$ and all $u_e^{J^P}$. The $u_e^{J^P}$ would then be over-determined by the $\phi_\omega^{J^P}$, whose staticity conditions, and those of the remaining $\phi_{{}^{(1)}\lambda}^{J^P}$, instead imply *secondary constraints* $\chi_\omega^{J^P} \approx \chi_{{}^{(1)}\lambda}^{J^P} \approx 0$ [43]. The conditions $\{\chi_X^{J^P}, \mathcal{H}'\} \approx 0$ determine most remaining $u_X^{J^P}$, but the tertiary $\zeta^{J^P}_{(1)\lambda}$ and quaternary $\theta^{J^P}_{(1)\lambda}$ constraints are required to determine them all. The lack of undetermined multipliers[4] is consistent with the absence of extra gauge symmetries expected for nonvanishing Proca mass in Eq. (4b).

In the nonlinear theory, many new commutators arise. Typically this would allow more $u^{J^P}_X$ to be determined by (5), requiring fewer $\chi^{J^P}_X$ and hence *emergent d.o.f in the nonlinear theory*. For example $L_{\rm EC}+\hat{\alpha}_5 R_{[\mu\nu]}R^{[\mu\nu]}$ propagates zero extra d.o.f when linearised, but *six* in general [20]. In our case, however, the abundant $\mathcal{O}(1)$ commutators already solve for the $u^{J^P}_X$ as efficiently as the possible pairings among the $\phi^{J^P}_X$ of each given J^P allow: exactly three of those six extra d.o.f — ${}^{(3)}T_\mu$ subject to $\mathring{\nabla}_\mu {}^{(3)}T^\mu = 0$ — are *always* present.

Effective field equations. – The constraint structure in Fig. 2 indicates greatly improved dynamical properties, compared to the usual higher-derivative extensions mentioned above. To access the phenomenology, we translate the resulting theory back into the familiar second order formalism of GR [38]. We will generalise back from $L_{\rm DC} + {}^{(2)}\lambda_\mu {}^{(2)}T^\mu$ to the full double-Proca case in (3). In the first order formalism [1] the $e^i{}_\mu$ field equation refers to the translational current

$$\tau^\mu{}_k(\omega) \equiv -\frac{\delta}{\delta e^k{}_\mu}\int \mathrm{d}^4x e L_{\rm M}(\omega), \tag{6}$$

in which $\omega^{ij}{}_\mu$ is held constant. Presuming that the matter Lagrangian is up to first order in $\omega^{ij}{}_\mu$, which is true in the case of Dirac matter, this can be partitioned

$$L_{\rm M}(\omega) = L_{\rm M}(\Delta) - \frac{1}{2}\sigma^\mu{}_{ij}\left(\omega^{ij}{}_\mu - \Delta^{ij}{}_\mu\right), \tag{7}$$

where all dependence on the torsion is concentrated into the second term, which is proportional to the spin tensor of matter

$$\sigma^\mu{}_{ij} \equiv -\frac{\delta}{\delta\omega^{ij}{}_\mu}\int \mathrm{d}^4x e L_{\rm M}(\omega), \tag{8}$$

and the Ricci rotation coefficients $\Delta^{ij}{}_\mu$ are the torsion-free limit of $\omega^{ij}{}_\mu$, depending only on $\partial_{[\mu}e^i{}_{\nu]}$. Using (7) and (6), the translational current can also be partitioned into $\tau^\mu{}_k(\omega) = \tau^\mu{}_k(\Delta) + \tau^\mu{}_k(\omega-\Delta)$. The second term is computed in terms of torsion and spin, given the (Dirac matter) dependence of $\sigma^\mu{}_{ij}$ on $e^i{}_\mu$. The first term is related to the Einstein stress-energy tensor [44]

$$T^\mu_\nu = \tau^\mu{}_\nu(\Delta) - \frac{1}{2}\mathring{\nabla}_\lambda\left(\sigma_\nu{}^{\mu\lambda} - \sigma^\lambda{}_\nu{}^\mu + \sigma^\mu{}_\nu{}^\lambda\right). \tag{9}$$

By projecting out the tensor part of the $\omega^{ij}{}_\mu$ field equation, the multiplier ${}^{(1)}\lambda^\lambda{}_{\mu\nu}$ can always be eliminated as an algebraically determined quantity. The antisymmetric part of the $e^i{}_\mu$ field equation is an identity, and by using (9) and separating the torsion from the Riemann–Cartan curvature[5] the symmetric part can eventually be shown to descend from the following effective theory

$$\begin{aligned}\mathcal{L}_{\rm T} \simeq \sqrt{-g}\Big[&-\frac{M_{\rm Pl}{}^2}{2}\mathring{R} + \frac{2\hat{\alpha}_5}{9}{}^{(2)}F_{\mu\nu}{}^{(2)}F^{\mu\nu} - \frac{\hat{\alpha}_5}{2}{}^{(3)}F_{\mu\nu}{}^{(3)}F^{\mu\nu} \\ &+ \frac{M_{\rm Pl}{}^2}{3}(1+2\hat{\beta}_2){}^{(2)}T_\mu{}^{(2)}T^\mu - \frac{3M_{\rm Pl}{}^2}{4}(1+8\hat{\beta}_3){}^{(3)}T_\mu{}^{(3)}T^\mu \\ &- \frac{1}{3}{}^{(2)}T_\mu{}^{(2)}S^\mu - \frac{3}{2}{}^{(3)}T_\mu{}^{(3)}S^\mu + L_{\rm M}\left(\mathring{\Gamma}\right)\Big]. \end{aligned}\tag{10}$$

In (10), the Maxwell terms are ${}^{(2)}F_{\mu\nu} \equiv 2\partial_{[\mu}{}^{(2)}T_{\nu]}$, etc., so that the origin of the Proca fields and their masses in Eqs. (4a) and (4b) is revealed. We define $S^\mu{}_{ij} \equiv \sigma^\mu{}_{ij}/e$. That these fields alone are propagating, and that they are sourced by the vector and pseudovector matter spin currents, may readily be confirmed by the parts of the $\omega^{ij}{}_\mu$ equation which do not serve to determine ${}^{(1)}\lambda^\lambda{}_{\mu\nu}$. The mandatory ghost-status of either one of the fields is also easy to read off from (10).

Einstein–Proca phenomenology. – [45]

Implications for fermion physics. – We expect the matter sector to contain various species Y of Dirac or Majorana fermions, from whose kinetic terms we can extract those parts which do not refer to internal gauge fields

$$\begin{aligned}L_{\rm M}(\Gamma) \supset \frac{i}{2}\sum_Y\Big[&\bar{\psi}_Y\left(1 - i\alpha - i\beta\gamma^5\right)\gamma^i e_i{}^\mu\nabla_\mu\psi_Y \\ &- \overline{\nabla_\mu\psi_Y}\left(1 + i\alpha + i\beta\gamma^5\right)\gamma^i e_i{}^\mu\psi_Y\Big].\end{aligned}\tag{11}$$

In (11) we follow [46] in allowing for non-minimal coupling through the universal, real parameters α and β [47–50]. It is important to accommodate for these extensions, since they yield different effective $\sim \bar{\psi}\psi\bar{\psi}\psi/M_{\rm Pl}{}^2$ interactions in the EC theory, and its natural Wilsonian extension in which the Barbaro–Immirzi parameter is finite. Based on (11) the spin sources in (10) are

$${}^{(2)}S^\mu = \sum_Y \frac{3}{2}\bar{\psi}_Y\left(\alpha + \beta\gamma^5\right)e_i{}^\mu\gamma^i\psi_Y, \tag{12a}$$

$${}^{(3)}S^\mu = -\sum_Y \frac{1}{2}\bar{\psi}_Y\gamma^5 e_i{}^\mu\gamma^i\psi_Y, \tag{12b}$$

so that ${}^{(2)}T_\mu$ and ${}^{(3)}T_\mu$ are sourced from the non-minimal and minimal sectors respectively.

Concluding remarks. –

[4] Our discussion assumes the presence of ten primary constraints of the super-momenta (which include $\mathcal{H}' \approx 0$), and their *undetermined* multipliers (which include the lapse and shift). These are basic features of the Poincaré gauge symmetry [38].

[5] Recall the standard formula $R \equiv \mathring{R} + \frac{8}{9}{}^{(1)}T_{\mu\nu\sigma}{}^{(1)}T^{\mu[\nu\sigma]} - \frac{2}{3}{}^{(2)}T_\mu{}^{(2)}T^\mu + \frac{2}{3}{}^{(3)}T_\mu{}^{(3)}T^\mu - 2\mathring{\nabla}_\mu{}^{(2)}T^\mu$ which gives rise to (2).

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