In[ ]:=
```
Get@FileNameJoin@{NotebookDirectory[], "Calibration.m"};
```

```
------------------------------------------------------------
Package xAct`xPerm`  version 1.2.3, {2015, 8, 23}
CopyRight (C) 2003-2020, Jose M. Martin-Garcia, under the General Public License.
Connecting to external linux executable...
Connection established.
------------------------------------------------------------
Package xAct`xTensor`  version 1.2.0, {2021, 10, 17}
CopyRight (C) 2002-2021, Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`xPlain` version 1.0.0-developer, {2023, 6, 10}
CopyRight © 2023, Will E. V. Barker and Sebastian Zell, under the General Public License.
------------------------------------------------------------
These packages come with ABSOLUTELY NO WARRANTY; for details type Disclaimer[]. This
  is free software, and you are welcome to redistribute it under certain conditions. See the General Public License for details.
------------------------------------------------------------
```

PSALTer Calibration

About xPlain and formatting

Welcome to the calibration file for the PSALTer package. Commentary is provided in this green text throughout by virtue of the xPlain package.

Key observation: Occasionally, more important points will be highlighted in boxes like this.

The xPlain package is not part of PSALTer, so the output from PSALTer itself will contrast with this formatting and be quite distinctive.

The structure of this file

The calibration file runs PSALTer on a very long list of theories, whose particle spectra are already known.

The first step is to load the PSALTer package.

```
------------------------------------------------------------
Package xAct`SymManipulator`  version 0.9.5, {2021, 9, 14}
CopyRight (C) 2011-2021, Thomas Bäckdahl, under the General Public License.
------------------------------------------------------------
Package xAct`xPert`  version 1.0.6, {2018, 2, 28}
CopyRight (C) 2005-2020, David Brizuela, Jose M. Martin-Garcia and Guillermo A. Mena Marugan, under the General Public License.
** Variable $CovDFormat changed from Prefix to Postfix
** Option AllowUpperDerivatives of ContractMetric changed from False to True
** Option MetricOn of MakeRule changed from None to All
** Option ContractMetrics of MakeRule changed from False to True
------------------------------------------------------------
Package xAct`Invar`  version 2.0.5, {2013, 7, 1}
CopyRight (C) 2006-2020, J. M. Martin-Garcia, D. Yllanes and R. Portugal, under the General Public License.
** DefConstantSymbol: Defining constant symbol sigma.
** DefConstantSymbol: Defining constant symbol dim.
** Option CurvatureRelations of DefCovD changed from True to False
** Variable $CommuteCovDsOnScalars changed from True to False
------------------------------------------------------------
Package xAct`xCoba`  version 0.8.6, {2021, 2, 28}
CopyRight (C) 2005-2021, David Yllanes and Jose M. Martin-Garcia, under the General Public License.
------------------------------------------------------------
Package xAct`xTras`  version 1.4.2, {2014, 10, 30}
CopyRight (C) 2012-2014, Teake Nutma, under the General Public License.
** Variable $CovDFormat changed from Postfix to Prefix
** Option CurvatureRelations of DefCovD changed from False to True
------------------------------------------------------------
Package xAct`PSALTer` version 1.0.0-developer, {2023, 8, 9}
CopyRight © 2022, Will E. V. Barker, Stephanie Buttigieg, Carlo Marzo, Cillian Rew, Claire Rigouzzo, Zhiyuan Wei and David Yallup, under the General Public License.
------------------------------------------------------------
These packages come with ABSOLUTELY NO WARRANTY; for details type Disclaimer[]. This
  is free software, and you are welcome to redistribute it under certain conditions. See the General Public License for details.
------------------------------------------------------------
Set: Tag ParticleSpectrum in Options[ParticleSpectrum] is Protected.
SetDelayed: Tag ParticleSpectrum in ParticleSpectrum[Expr_, OptionsPattern[]] is Protected.
```

Great, so PSALTer is now loaded and we can start to do some science.

Poincaré gauge theory (PGT)

Key observation: We will test the PoincareGaugeTheory module.

Here is the inverse translational gauge field, or tetrad.

$$h_{\alpha}{}^{\chi} \tag{1}$$

Here is the translational gauge field, or inverse tetrad.

$$b^{\alpha}{}_{\chi} \tag{2}$$

Here is the Riemann-Cartan tensor.

$$\mathcal{R}^{\alpha\beta}{}_{\delta\epsilon} \tag{3}$$

$$\mathcal{A}^{\alpha\gamma}{}_{\phi}\ \mathcal{A}^{\beta}{}_{\gamma\chi}\ h_{\delta}{}^{\chi}\ h_{\epsilon}{}^{\phi} - \mathcal{A}^{\alpha\gamma}{}_{\chi}\ \mathcal{A}^{\beta}{}_{\gamma\phi}\ h_{\delta}{}^{\chi}\ h_{\epsilon}{}^{\phi} + h_{\delta}{}^{\chi}\ h_{\epsilon}{}^{\phi}\ \partial_{\chi}\mathcal{A}^{\alpha\beta}{}_{\phi} - h_{\delta}{}^{\chi}\ h_{\epsilon}{}^{\phi}\ \partial_{\phi}\mathcal{A}^{\alpha\beta}{}_{\chi} \tag{4}$$

Here is the torsion tensor.

$$\mathcal{T}^{\alpha}{}_{\beta\chi} \tag{5}$$

$$\mathcal{A}^{\alpha}{}_{\chi\delta}\ h_{\beta}{}^{\delta} - \mathcal{A}^{\alpha}{}_{\beta\delta}\ h_{\chi}{}^{\delta} + h_{\beta}{}^{\delta}\ h_{\chi}{}^{\epsilon}\ \partial_{\delta}b^{\alpha}{}_{\epsilon} - h_{\beta}{}^{\delta}\ h_{\chi}{}^{\epsilon}\ \partial_{\epsilon}b^{\alpha}{}_{\delta} \tag{6}$$

Now we set up the general Lagrangian. In the first instance we will do this with some coupling constants which are proportional to those used by Hayashi and Shirafuji in Prog. Theor. Phys. 64 (1980) 2222. The normalisations are not absolutely identical, but this should not be a problem.

$$-\frac{1}{2}\alpha_{0}\ \eta^{\alpha\chi}\ \eta^{\beta\delta}\ \mathcal{R}_{\alpha\beta\chi\delta} + \left(\alpha_{1}\ \hat{\mathcal{P}}_{\mathcal{R}}1_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{2}\ \hat{\mathcal{P}}_{\mathcal{R}}2_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{3}\ \hat{\mathcal{P}}_{\mathcal{R}}3_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{4}\ \hat{\mathcal{P}}_{\mathcal{R}}4_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{5}\ \hat{\mathcal{P}}_{\mathcal{R}}5_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta} + \alpha_{6}\ \hat{\mathcal{P}}_{\mathcal{R}}6_{\iota\theta\gamma\eta}{}^{\alpha\beta\chi\delta}\right)\mathcal{R}_{\alpha\beta\chi\delta}\ \mathcal{R}^{\iota\theta\gamma\eta} + \left(\beta_{1}\ \hat{\mathcal{P}}_{\mathcal{T}}1_{\iota\gamma\eta}{}^{\alpha\chi\delta} + \beta_{2}\ \hat{\mathcal{P}}_{\mathcal{T}}2_{\iota\gamma\eta}{}^{\alpha\chi\delta} + \beta_{3}\ \hat{\mathcal{P}}_{\mathcal{T}}3_{\iota\gamma\eta}{}^{\alpha\chi\delta}\right)\mathcal{T}_{\alpha\chi\delta}\ \mathcal{T}^{\iota\gamma\eta} \tag{7}$$

In Eq. (7) we are using projectors to extract the Lorentz irreps of the fields. Next we will expand these.

So with the projectors expanded we have the following nonlinear Lagrangian.

$$-\frac{1}{2}\alpha_{0}\ \mathcal{R}^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{6}\left(2\alpha_{1} + 3\alpha_{2} + \alpha_{3}\right)\mathcal{R}_{\alpha\beta\chi\delta}\ \mathcal{R}^{\alpha\beta\chi\delta} + \frac{2}{3}\left(\alpha_{1} - \alpha_{3}\right)\mathcal{R}_{\alpha\chi\beta\delta}\ \mathcal{R}^{\alpha\beta\chi\delta} + \left(-\alpha_{1} - \alpha_{2} + \alpha_{4} + \alpha_{5}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha}{}^{\chi}\ \mathcal{R}_{\beta}{}^{\delta}{}_{\chi\delta} + \frac{1}{6}\left(2\alpha_{1} - 3\alpha_{2} + \alpha_{3}\right)\mathcal{R}^{\alpha\beta\chi\delta}\ \mathcal{R}_{\chi\delta\alpha\beta} + \left(-\alpha_{1} + \alpha_{2} + \alpha_{4} - \alpha_{5}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha}{}^{\chi}\ \mathcal{R}_{\chi}{}^{\delta}{}_{\beta\delta} + \frac{1}{6}\left(2\alpha_{1} - 3\alpha_{4} + \alpha_{6}\right)\mathcal{R}^{\alpha\beta}{}_{\alpha\beta}\ \mathcal{R}^{\chi\delta}{}_{\chi\delta} + \frac{1}{3}\left(2\beta_{1} + \beta_{3}\right)\mathcal{T}_{\alpha\beta\chi}\ \mathcal{T}^{\alpha\beta\chi} + \frac{2}{3}\left(\beta_{1} - \beta_{3}\right)\mathcal{T}^{\alpha\beta\chi}\ \mathcal{T}_{\beta\alpha\chi} + \frac{2}{3}\left(\beta_{1} - \beta_{2}\right)\mathcal{T}^{\alpha}{}_{\alpha}{}^{\beta}\ \mathcal{T}^{\chi}{}_{\beta\chi} \tag{8}$$

We can also use a different set of coupling coefficients, as developed by Karananas.

$$-\lambda\ \mathcal{R}^{\iota\theta}{}_{\iota\theta} + \left(\frac{r_{1}}{3} + \frac{r_{2}}{6}\right)\mathcal{R}_{\iota\theta\kappa\lambda}\ \mathcal{R}^{\iota\theta\kappa\lambda} + \left(\frac{2r_{1}}{3} - \frac{2r_{2}}{3}\right)\mathcal{R}_{\iota\kappa\theta\lambda}\ \mathcal{R}^{\iota\theta\kappa\lambda} + \left(r_{4} + r_{5}\right)\mathcal{R}_{\iota}{}^{\lambda}{}_{\theta\lambda}\ \mathcal{R}^{\iota\kappa\theta}{}_{\kappa} + \tag{9}$$

$$\left(r_4 - r_5\right) \mathcal{R}^{\iota\kappa\theta}{}_{\kappa}\, \mathcal{R}_{\theta}{}^{\lambda}{}_{\iota\lambda} + \left(\frac{r_1}{3} + \frac{r_2}{6} - r_3\right) \mathcal{R}^{\iota\theta\kappa\lambda}\, \mathcal{R}_{\kappa\lambda\iota\theta} + \left(\frac{\lambda}{4} + \frac{t_1}{3} + \frac{t_2}{12}\right) \mathcal{T}_{\iota\theta\kappa}\, \mathcal{T}^{\iota\theta\kappa} + \left(-\frac{\lambda}{2} - \frac{t_1}{3} + \frac{t_2}{6}\right) \mathcal{T}^{\iota\theta\kappa}\, \mathcal{T}_{\theta\kappa\iota} + \left(-\lambda - \frac{t_1}{3} + \frac{2 t_3}{3}\right) \mathcal{T}_{\iota}{}^{\theta\iota}\, \mathcal{T}_{\kappa\theta}{}^{\kappa}$$

Minimal even-parity scalar model

We will study the minimal model set out in Eq. (4.1) of arXiv:9902032. We will do this using the general coupling coefficients defined in Eq. (8).

$$-\frac{1}{2}\alpha_0\, \mathcal{R}^{\alpha\beta}{}_{\alpha\beta} + \frac{1}{6}\alpha_6\, \mathcal{R}^{\alpha\beta}{}_{\alpha\beta}\, \mathcal{R}^{\chi\delta}{}_{\chi\delta} + \frac{1}{2}\beta_1\, \mathcal{T}_{\alpha\beta\chi}\, \mathcal{T}^{\alpha\beta\chi} + \beta_1\, \mathcal{T}^{\alpha\beta\chi}\, \mathcal{T}_{\beta\alpha\chi} + 2\beta_1\, \mathcal{T}^{\alpha}{}_{\alpha}{}^{\beta}\, \mathcal{T}^{\chi}{}_{\beta\chi} \tag{10}$$

PSALTer results panel

$$\mathcal{S} == \iiiint \Big(-\tfrac{1}{2}\left(\alpha_0 - 4\beta_1\right) \mathcal{A}^{\alpha\beta}{}_{\alpha}\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi} + \mathcal{A}^{\alpha\beta\chi}\, \sigma_{\alpha\beta\chi} + f^{\alpha\beta}\, \tau\left(\Delta+\mathcal{K}\right)_{\alpha\beta} - \alpha_0\, f^{\alpha\beta}\, \partial_{\beta}\mathcal{A}_{\alpha}{}^{\chi}{}_{\chi} + \alpha_0\, \partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha} - 4\beta_1\, \mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}\, \partial_{\beta}f^{\alpha\beta} + 4\beta_1\, \mathcal{A}_{\beta}{}^{\chi}{}_{\chi}\, \partial^{\beta}f^{\alpha}{}_{\alpha} - 2\beta_1\, \partial_{\beta}f^{\chi}{}_{\chi}\, \partial^{\beta}f^{\alpha}{}_{\alpha} + \alpha_0\, f^{\alpha\beta}\, \partial_{\chi}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta} - \alpha_0\, f^{\alpha}{}_{\alpha}\, \partial_{\chi}\mathcal{A}^{\beta\chi}{}_{\beta} - 2\beta_1\, \partial_{\beta}f^{\alpha\beta}\, \partial_{\chi}f_{\alpha}{}^{\chi} + 4\beta_1\, \partial^{\beta}f^{\alpha}{}_{\alpha}\, \partial_{\chi}f_{\beta}{}^{\chi} - 2\beta_1\, \partial_{\alpha}f_{\beta\chi}\, \partial^{\chi}f^{\alpha\beta} - \beta_1\, \partial_{\alpha}f_{\chi\beta}\, \partial^{\chi}f^{\alpha\beta} + \beta_1\, \partial_{\beta}f_{\alpha\chi}\, \partial^{\chi}f^{\alpha\beta} + \beta_1\, \partial_{\chi}f_{\alpha\beta}\, \partial^{\chi}f^{\alpha\beta} + \beta_1\, \partial_{\chi}f_{\beta\alpha}\, \partial^{\chi}f^{\alpha\beta} - \tfrac{1}{2}\mathcal{A}_{\alpha\chi\beta}\left(\left(\alpha_0 - 4\beta_1\right)\mathcal{A}^{\alpha\beta\chi} - 8\beta_1\, \partial^{\chi}f^{\alpha\beta}\right) + \tfrac{2}{3}\alpha_6\, \partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha}\, \partial_{\delta}\mathcal{A}^{\chi\delta}{}_{\chi}\Big)[t,x,y,z]\, dz\, dy\, dx\, dt$$

Wave operator

	${}^{0^+}\mathcal{A}^{\parallel}$	${}^{0^+}f^{\parallel}$	${}^{0^+}f^{\perp}$	${}^{0^-}\mathcal{A}^{\parallel}$
${}^{0^+}\mathcal{A}^{\parallel}\dagger$	$\frac{\alpha_0}{2} - 2\beta_1 + \alpha_6 k^2$	$-\frac{i\left(\alpha_0-4\beta_1\right)k}{\sqrt{2}}$	0	0
${}^{0^+}f^{\parallel}\dagger$	$\frac{i\left(\alpha_0-4\beta_1\right)k}{\sqrt{2}}$	$-4\beta_1 k^2$	0	0
${}^{0^+}f^{\perp}\dagger$	0	0	0	0
${}^{0^-}\mathcal{A}^{\parallel}\dagger$	0	0	0	$\frac{1}{2}\left(\alpha_0 - 4\beta_1\right)$

	${}^{1^+}\mathcal{A}^{\parallel}{}_{\alpha\beta}$	${}^{1^+}\mathcal{A}^{\perp}{}_{\alpha\beta}$	${}^{1^+}f^{\parallel}{}_{\alpha\beta}$	${}^{1^-}\mathcal{A}^{\parallel}{}_{\alpha}$	${}^{1^-}\mathcal{A}^{\perp}{}_{\alpha}$	${}^{1^-}f^{\parallel}{}_{\alpha}$	${}^{1^-}f^{\perp}{}_{\alpha}$
${}^{1^+}\mathcal{A}^{\parallel}\dagger^{\alpha\beta}$	$\frac{1}{4}\left(\alpha_0 - 4\beta_1\right)$	$\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	$\frac{i\left(\alpha_0-4\beta_1\right)k}{2\sqrt{2}}$	0	0	0	0
${}^{1^+}\mathcal{A}^{\perp}\dagger^{\alpha\beta}$	$\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	0	0	0	0	0	0
${}^{1^+}f^{\parallel}\dagger^{\alpha\beta}$	$-\frac{i\left(\alpha_0-4\beta_1\right)k}{2\sqrt{2}}$	0	0	0	0	0	0
${}^{1^-}\mathcal{A}^{\parallel}\dagger^{\alpha}$	0	0	0	$\frac{1}{4}\left(\alpha_0 - 4\beta_1\right)$	$-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	0	$-\frac{1}{2} i\left(\alpha_0 - 4\beta_1\right)k$
${}^{1^-}\mathcal{A}^{\perp}\dagger^{\alpha}$	0	0	0	$-\frac{\alpha_0-4\beta_1}{2\sqrt{2}}$	0	0	0
${}^{1^-}f^{\parallel}\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}f^{\perp}\dagger^{\alpha}$	0	0	0	$\frac{1}{2} i\left(\alpha_0 - 4\beta_1\right)k$	0	0	0

	${}^{2^+}\mathcal{A}^{\parallel}{}_{\alpha\beta}$	${}^{2^+}f^{\parallel}{}_{\alpha\beta}$	${}^{2^-}\mathcal{A}^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^+}\mathcal{A}^{\parallel}\dagger^{\alpha\beta}$	$-\frac{\alpha_0}{4} + \beta_1$	$\frac{i\left(\alpha_0-4\beta_1\right)k}{2\sqrt{2}}$	0
${}^{2^+}f^{\parallel}\dagger^{\alpha\beta}$	$-\frac{i\left(\alpha_0-4\beta_1\right)k}{2\sqrt{2}}$	$2\beta_1 k^2$	0
${}^{2^-}\mathcal{A}^{\parallel}\dagger^{\alpha\beta\chi}$	0	0	$-\frac{\alpha_0}{4} + \beta_1$

Saturated propagator

	${}^{0^+}_{\cdot}\sigma^{\parallel}$	${}^{0^+}_{\cdot}\tau^{\parallel}$	${}^{0^+}_{\cdot}\tau^{\perp}$	${}^{0^-}_{\cdot}\sigma^{\parallel}$
${}^{0^+}_{\cdot}\sigma^{\parallel}\,\dagger$	$\frac{8\beta_{\dot 1}}{\alpha_{\dot 0}^2-4\alpha_{\dot 0}\beta_{\dot 1}+8\alpha_{\dot 6}\beta_{\dot 1}k^2}$	$-\frac{i\sqrt{2}\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)}{\alpha_{\dot 0}\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)k+8\alpha_{\dot 6}\beta_{\dot 1}k^3}$	0	0
${}^{0^+}_{\cdot}\tau^{\parallel}\,\dagger$	$\frac{i\sqrt{2}\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)}{\alpha_{\dot 0}\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)k+8\alpha_{\dot 6}\beta_{\dot 1}k^3}$	$-\frac{\alpha_{\dot 0}-4\beta_{\dot 1}+2\alpha_{\dot 6}k^2}{k^2\left(\alpha_{\dot 0}^2-4\alpha_{\dot 0}\beta_{\dot 1}+8\alpha_{\dot 6}\beta_{\dot 1}k^2\right)}$	0	0
${}^{0^+}_{\cdot}\tau^{\perp}\,\dagger$	0	0	0	0
${}^{0^-}_{\cdot}\sigma^{\parallel}\,\dagger$	0	0	0	$\frac{2}{\alpha_{\dot 0}-4\beta_{\dot 1}}$

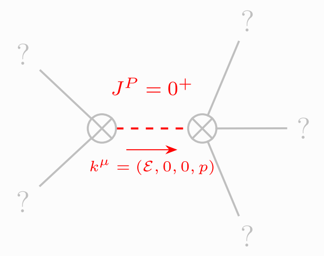
	${}^{1^+}_{\cdot}\sigma^{\parallel}{}_{\alpha\beta}$	${}^{1^+}_{\cdot}\sigma^{\perp}{}_{\alpha\beta}$	${}^{1^+}_{\cdot}\tau^{\parallel}{}_{\alpha\beta}$	${}^{1^-}_{\cdot}\sigma^{\parallel}{}_{\alpha}$	${}^{1^-}_{\cdot}\sigma^{\perp}{}_{\alpha}$	${}^{1^-}_{\cdot}\tau^{\parallel}{}_{\alpha}$	${}^{1^-}_{\cdot}\tau^{\perp}{}_{\alpha}$
${}^{1^+}_{\cdot}\sigma^{\parallel}\,\dagger^{\alpha\beta}$	0	$\frac{2\sqrt{2}}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)}$	$\frac{2i\sqrt{2}\,k}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)}$	0	0	0	0
${}^{1^+}_{\cdot}\sigma^{\perp}\,\dagger^{\alpha\beta}$	$\frac{2\sqrt{2}}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)}$	$-\frac{2}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)^2}$	$-\frac{2ik}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)^2}$	0	0	0	0
${}^{1^+}_{\cdot}\tau^{\parallel}\,\dagger^{\alpha\beta}$	$-\frac{2i\sqrt{2}\,k}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)}$	$\frac{2ik}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)^2}$	$-\frac{2k^2}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+k^2)^2}$	0	0	0	0
${}^{1^-}_{\cdot}\sigma^{\parallel}\,\dagger^{\alpha}$	0	0	0	0	$-\frac{2\sqrt{2}}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)}$	0	$-\frac{4ik}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)}$
${}^{1^-}_{\cdot}\sigma^{\perp}\,\dagger^{\alpha}$	0	0	0	$-\frac{2\sqrt{2}}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)}$	$-\frac{2}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)^2}$	0	$-\frac{2i\sqrt{2}\,k}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)^2}$
${}^{1^-}_{\cdot}\tau^{\parallel}\,\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}_{\cdot}\tau^{\perp}\,\dagger^{\alpha}$	0	0	0	$\frac{4ik}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)}$	$\frac{2i\sqrt{2}\,k}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)^2}$	0	$-\frac{4k^2}{\left(\alpha_{\dot 0}-4\beta_{\dot 1}\right)(1+2k^2)^2}$

	${}^{2^+}_{\cdot}\sigma^{\parallel}{}_{\alpha\beta}$	${}^{2^+}_{\cdot}\tau^{\parallel}{}_{\alpha\beta}$	${}^{2^-}_{\cdot}\sigma^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^+}_{\cdot}\sigma^{\parallel}\,\dagger^{\alpha\beta}$	$-\frac{16\beta_{\dot 1}}{\alpha_{\dot 0}^2-4\alpha_{\dot 0}\beta_{\dot 1}}$	$\frac{2i\sqrt{2}}{\alpha_{\dot 0}k}$	0
${}^{2^+}_{\cdot}\tau^{\parallel}\,\dagger^{\alpha\beta}$	$-\frac{2i\sqrt{2}}{\alpha_{\dot 0}k}$	$\frac{2}{\alpha_{\dot 0}k^2}$	0
${}^{2^-}_{\cdot}\sigma^{\parallel}\,\dagger^{\alpha\beta\chi}$	0	0	$\frac{1}{-\frac{\alpha_{\dot 0}}{4}+\beta_{\dot 1}}$

Source constraints

Spin-parity form	Covariant form	Multiplicities
${}^{0^+}_{\cdot}\tau^{\perp} == 0$	$\partial_\beta\partial_\alpha\tau\,(\Delta+\mathcal{K})^{\alpha\beta} == 0$	1
$2\,i\,k\;{}^{1^-}_{\cdot}\sigma^{\perp\,\alpha} + {}^{1^-}_{\cdot}\tau^{\perp\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau\,(\Delta+\mathcal{K})^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\beta\alpha\chi}$	3
${}^{1^-}_{\cdot}\tau^{\parallel\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau\,(\Delta+\mathcal{K})^{\beta\alpha}$	3
$i\,k\;{}^{1^+}_{\cdot}\sigma^{\perp\,\alpha\beta} + {}^{1^+}_{\cdot}\tau^{\parallel\,\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\beta\chi} + \partial_\chi\partial^\beta\tau\,(\Delta+\mathcal{K})^{\chi\alpha} + \partial_\chi\partial^\chi\tau\,(\Delta+\mathcal{K})^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\chi\beta\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\chi\alpha\beta} == \partial_\chi\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\chi\beta} + \partial_\chi\partial^\beta\tau\,(\Delta+\mathcal{K})^{\alpha\chi} + \partial_\chi\partial^\chi\tau\,(\Delta+\mathcal{K})^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\chi\alpha\delta}$	3
Total expected gauge generators:		10

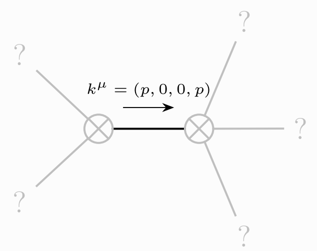
Massive spectrum



Massive particle

Pole residue:	$\frac{1}{\alpha_{0}} + \frac{1}{\alpha_{6}} - \frac{1}{4\beta_{1}} > 0$
Square mass:	$-\frac{\alpha_{0}(\alpha_{0}-4\beta_{1})}{8\alpha_{6}\beta_{1}} > 0$
Spin:	0
Parity:	Even

Massless spectrum



Massless particle

Pole residue:	$\frac{p^2}{\alpha_{0}} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented in PSALTer)

Unitarity conditions

$\alpha_{0} > 0 \,\&\&\, \alpha_{6} > 0 \,\&\&\, \left(\beta_{1} < 0 \,||\, \beta_{1} > \frac{\alpha_{0}}{4}\right)$

Validity assumptions

(Not yet implemented in PSALTer)

Key observation: Thus we see that only the odd-parity scalar mode is moving with a mass, as claimed.

Minimal massive odd-parity scalar model

We will study the minimal model set out in Eq. (4.25) of arXiv:9902032. We will do this using the general coupling coefficients defined in Eq. (8).

$$-\frac{1}{2}\alpha_{0}\,\mathcal{R}^{\alpha\beta}{}_{\alpha\beta}+\frac{1}{6}\alpha_{3}\,\mathcal{R}_{\alpha\beta\chi\delta}\,\mathcal{R}^{\alpha\beta\chi\delta}-\frac{2}{3}\alpha_{3}\,\mathcal{R}_{\alpha\chi\beta\delta}\,\mathcal{R}^{\alpha\beta\chi\delta}+\frac{1}{6}\alpha_{3}\,\mathcal{R}^{\alpha\beta\chi\delta}\,\mathcal{R}_{\chi\delta\alpha\beta}+\frac{1}{2}\beta_{1}\,\mathcal{T}_{\alpha\beta\chi}\,\mathcal{T}^{\alpha\beta\chi}+\beta_{1}\,\mathcal{T}^{\alpha\beta\chi}\,\mathcal{T}_{\beta\alpha\chi}+2\beta_{1}\,\mathcal{T}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}^{\chi}{}_{\beta\chi} \qquad (11)$$

PSALTer results panel

$$\mathcal{S} == \iiiint\Big(\mathcal{A}^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}+f^{\alpha\beta}\,\tau(\Delta+\mathcal{K})_{\alpha\beta}--\alpha.\Big(\mathcal{A}_{\alpha\chi\beta}\,\mathcal{A}^{\alpha\beta\chi}+\mathcal{A}^{\alpha\beta}{}_{\alpha}\,\mathcal{A}_{\beta}{}^{\chi}{}_{\chi}+2f^{\alpha\beta}\,\partial_{\beta}\mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}-2\,\partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha}-2f^{\alpha\beta}\,\partial_{\chi}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}+2f^{\alpha}{}_{\alpha}\,\partial_{\chi}\mathcal{A}^{\beta\chi}{}_{\beta}\Big)+\beta.\Big(2\mathcal{A}^{\alpha\beta}{}_{\alpha}\,\mathcal{A}_{\beta}{}^{\chi}{}_{\chi}-4\mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}\,\partial_{\beta}f^{\alpha\beta}+4\mathcal{A}_{\beta}{}^{\chi}{}_{\chi}\,\partial^{\beta}f^{\alpha}{}_{\alpha}-2\,\partial_{\beta}f^{\chi}{}_{\chi}\,\partial^{\beta}f^{\alpha}{}_{\alpha}-2\,\partial_{\beta}f^{\alpha\beta}\,\partial_{\chi}f_{\alpha}{}^{\chi}+4\,\partial^{\beta}f^{\alpha}{}_{\alpha}\,\partial_{\chi}f_{\beta}{}^{\chi}-2\,\partial_{\alpha}f_{\beta\chi}\,\partial^{\chi}f^{\alpha\beta}-\partial_{\alpha}f_{\chi\beta}\,\partial^{\chi}f^{\alpha\beta}+\partial_{\beta}f_{\alpha\chi}\,\partial^{\chi}f^{\alpha\beta}+\partial_{\chi}f_{\alpha\beta}\,\partial^{\chi}f^{\alpha\beta}+\partial_{\chi}f_{\beta\alpha}\,\partial^{\chi}f^{\alpha\beta}+2\mathcal{A}_{\alpha\chi\beta}\big(\mathcal{A}^{\alpha\beta\chi}+2\,\partial^{\chi}f^{\alpha\beta}\big)\Big)+-\alpha.\Big(4\,\partial_{\beta}\mathcal{A}_{\alpha\chi\delta}-2\,\partial_{\beta}\mathcal{A}_{\alpha\delta\chi}+2\,\partial_{\beta}\mathcal{A}_{\chi\delta\alpha}-\partial_{\chi}\mathcal{A}_{\alpha\beta\delta}+\partial_{\delta}\mathcal{A}_{\alpha\beta\chi}-2\,\partial_{\delta}\mathcal{A}_{\alpha\chi\beta}\Big)\partial^{\delta}\mathcal{A}^{\alpha\beta\chi}\Big)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Wave operator

	${}^{0^+}\mathcal{A}^{\parallel}$	${}^{0^+}f^{\parallel}$	${}^{0^+}f^{\perp}$	${}^{0^-}\mathcal{A}^{\parallel}$
${}^{0^+}\mathcal{A}^{\parallel}\dagger$	$-(\alpha.-4\beta.)$	$-\frac{i(\alpha_{0}-4\beta_{1})k}{\sqrt{2}}$	0	0
${}^{0^+}f^{\parallel}\dagger$	$\frac{i(\alpha_{0}-4\beta_{1})k}{\sqrt{2}}$	$-4\beta_{1}k^2$	0	0
${}^{0^+}f^{\perp}\dagger$	0	0	0	0
${}^{0^-}\mathcal{A}^{\parallel}\dagger$	0	0	0	$\frac{\alpha_{0}}{2}-2\beta_{1}+\alpha_{3}k^2$

	${}^{1^+}\mathcal{A}^{\parallel}{}_{\alpha\beta}$	${}^{1^+}\mathcal{A}^{\perp}{}_{\alpha\beta}$	${}^{1^+}f^{\parallel}{}_{\alpha\beta}$	${}^{1^-}\mathcal{A}^{\parallel}{}_{\alpha}$	${}^{1^-}\mathcal{A}^{\perp}{}_{\alpha}$	${}^{1^-}f^{\parallel}{}_{\alpha}$	${}^{1^-}f^{\perp}{}_{\alpha}$
${}^{1^+}\mathcal{A}^{\parallel}\dagger^{\alpha\beta}$	$\frac{1}{4}(\alpha_{0}-4\beta_{1})$	$\frac{\alpha_{0}-4\beta_{1}}{2\sqrt{2}}$	$\frac{i(\alpha_{0}-4\beta_{1})k}{2\sqrt{2}}$	0	0	0	0
${}^{1^+}\mathcal{A}^{\perp}\dagger^{\alpha\beta}$	$\frac{\alpha_{0}-4\beta_{1}}{2\sqrt{2}}$	0	0	0	0	0	0
${}^{1^+}f^{\parallel}\dagger^{\alpha\beta}$	$-\frac{i(\alpha_{0}-4\beta_{1})k}{2\sqrt{2}}$	0	0	0	0	0	0
${}^{1^-}\mathcal{A}^{\parallel}\dagger^{\alpha}$	0	0	0	$\frac{1}{4}(\alpha_{0}-4\beta_{1})$	$-\frac{\alpha_{0}-4\beta_{1}}{2\sqrt{2}}$	0	$-\frac{1}{2}i(\alpha_{0}-4\beta_{1})k$
${}^{1^-}\mathcal{A}^{\perp}\dagger^{\alpha}$	0	0	0	$-\frac{\alpha_{0}-4\beta_{1}}{2\sqrt{2}}$	0	0	0
${}^{1^-}f^{\parallel}\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}f^{\perp}\dagger^{\alpha}$	0	0	0	$\frac{1}{2}i(\alpha_{0}-4\beta_{1})k$	0	0	0

	${}^{2^+}\mathcal{A}^{\parallel}{}_{\alpha\beta}$	${}^{2^+}f^{\parallel}{}_{\alpha\beta}$	${}^{2^-}\mathcal{A}^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^+}\mathcal{A}^{\parallel}\dagger^{\alpha\beta}$	$-\frac{\alpha_{0}}{4}+\beta_{1}$	$\frac{i(\alpha_{0}-4\beta_{1})k}{2\sqrt{2}}$	0
${}^{2^+}f^{\parallel}\dagger^{\alpha\beta}$	$-\frac{i(\alpha_{0}-4\beta_{1})k}{2\sqrt{2}}$	$2\beta_{1}k^2$	0
${}^{2^-}\mathcal{A}^{\parallel}\dagger^{\alpha\beta\chi}$	0	0	$-\frac{\alpha_{0}}{4}+\beta_{1}$

Saturated propagator

	${}^{0^+}\sigma^{\parallel}$	${}^{0^+}\tau^{\parallel}$	${}^{0^+}\tau^{\perp}$	${}^{0^-}\sigma^{\parallel}$
${}^{0^+}\sigma^{\parallel}\dagger$	$\frac{8\beta_1}{\alpha_0^2-4\alpha_0\beta_1}$	$-\frac{i\sqrt{2}}{\alpha_0 k}$	0	0
${}^{0^+}\tau^{\parallel}\dagger$	$\frac{i\sqrt{2}}{\alpha_0 k}$	$-\frac{1}{\alpha_0 k^2}$	0	0
${}^{0^+}\tau^{\perp}\dagger$	0	0	0	0
${}^{0^-}\sigma^{\parallel}\dagger$	0	0	0	$\frac{2}{\alpha_0-4\beta_1+2\alpha_3 k^2}$

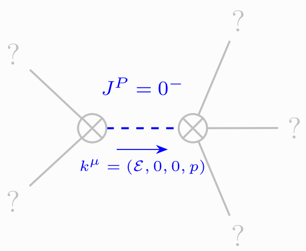
	${}^{1^+}\sigma^{\parallel}{}_{\alpha\beta}$	${}^{1^+}\sigma^{\perp}{}_{\alpha\beta}$	${}^{1^+}\tau^{\parallel}{}_{\alpha\beta}$	${}^{1^-}\sigma^{\parallel}{}_{\alpha}$	${}^{1^-}\sigma^{\perp}{}_{\alpha}$	${}^{1^-}\tau^{\parallel}{}_{\alpha}$	${}^{1^-}\tau^{\perp}{}_{\alpha}$
${}^{1^+}\sigma^{\parallel}\dagger^{\alpha\beta}$	0	$\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$	$\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$	0	0	0	0
${}^{1^+}\sigma^{\perp}\dagger^{\alpha\beta}$	$\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+k^2)}$	$-\frac{2}{(\alpha_0-4\beta_1)(1+k^2)^2}$	$-\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$	0	0	0	0
${}^{1^+}\tau^{\parallel}\dagger^{\alpha\beta}$	$-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+k^2)}$	$\frac{2ik}{(\alpha_0-4\beta_1)(1+k^2)^2}$	$-\frac{2k^2}{(\alpha_0-4\beta_1)(1+k^2)^2}$	0	0	0	0
${}^{1^-}\sigma^{\parallel}\dagger^{\alpha}$	0	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$	0	$-\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$
${}^{1^-}\sigma^{\perp}\dagger^{\alpha}$	0	0	0	$-\frac{2\sqrt{2}}{(\alpha_0-4\beta_1)(1+2k^2)}$	$-\frac{2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$	0	$-\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$
${}^{1^-}\tau^{\parallel}\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}\tau^{\perp}\dagger^{\alpha}$	0	0	0	$\frac{4ik}{(\alpha_0-4\beta_1)(1+2k^2)}$	$\frac{2i\sqrt{2}k}{(\alpha_0-4\beta_1)(1+2k^2)^2}$	0	$-\frac{4k^2}{(\alpha_0-4\beta_1)(1+2k^2)^2}$

	${}^{2^+}\sigma^{\parallel}{}_{\alpha\beta}$	${}^{2^+}\tau^{\parallel}{}_{\alpha\beta}$	${}^{2^-}\sigma^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^+}\sigma^{\parallel}\dagger^{\alpha\beta}$	$-\frac{16\beta_1}{\alpha_0^2-4\alpha_0\beta_1}$	$\frac{2i\sqrt{2}}{\alpha_0 k}$	0
${}^{2^+}\tau^{\parallel}\dagger^{\alpha\beta}$	$-\frac{2i\sqrt{2}}{\alpha_0 k}$	$\frac{2}{\alpha_0 k^2}$	0
${}^{2^-}\sigma^{\parallel}\dagger^{\alpha\beta\chi}$	0	0	$\frac{1}{-\frac{\alpha_0}{4}+\beta_1}$

Source constraints

Spin-parity form	Covariant form	Multiplicities
${}^{0^+}\tau^{\perp} == 0$	$\partial_\beta\partial_\alpha\tau(\Delta+\mathcal{K})^{\alpha\beta} == 0$	1
$2ik\,{}^{1^-}\sigma^{\perp\alpha} + {}^{1^-}\tau^{\perp\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau(\Delta+\mathcal{K})^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau(\Delta+\mathcal{K})^{\alpha\beta} + 2\,\partial_\delta\partial^\delta\partial_\chi\partial_\beta\sigma^{\beta\alpha\chi}$	3
${}^{1^-}\tau^{\parallel\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau(\Delta+\mathcal{K})^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau(\Delta+\mathcal{K})^{\beta\alpha}$	3
$ik\,{}^{1^+}\sigma^{\perp\alpha\beta} + {}^{1^+}\tau^{\parallel\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau(\Delta+\mathcal{K})^{\beta\chi} + \partial_\chi\partial^\beta\tau(\Delta+\mathcal{K})^{\chi\alpha} + \partial_\chi\partial^\chi\tau(\Delta+\mathcal{K})^{\alpha\beta} + 2\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\chi\beta\delta} + 2\,\partial_\delta\partial^\delta\partial_\chi\sigma^{\chi\alpha\beta} == \partial_\chi\partial^\alpha\tau(\Delta+\mathcal{K})^{\chi\beta} + \partial_\chi\partial^\beta\tau(\Delta+\mathcal{K})^{\alpha\chi} + \partial_\chi\partial^\chi\tau(\Delta+\mathcal{K})^{\beta\alpha} + 2\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\chi\alpha\delta}$	3
Total expected gauge generators:		10

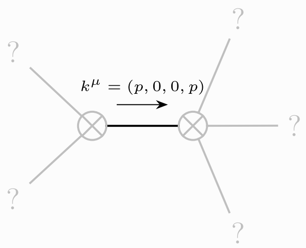
Massive spectrum



Massive particle

Pole residue:	$-\frac{1}{\alpha_3} > 0$
Square mass:	$-\frac{\alpha_0 - 4\beta_1}{2\alpha_3} > 0$
Spin:	0
Parity:	Odd

Massless spectrum



Massless particle

Pole residue:	$\frac{p^2}{\alpha_0} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented in PSALTer)

Unitarity conditions

$\alpha_0 > 0 \;\&\&\; \alpha_3 < 0 \;\&\&\; \beta_1 < \frac{\alpha_0}{4}$

Validity assumptions

(Not yet implemented in PSALTer)

Key observation: Thus we see that only the even-parity scalar mode is moving with a mass, as claimed.

Minimal massless odd-parity scalar model

We will study the minimal model set out between Eqs. (4.47) and (4.48) of arXiv:9902032. We will do this using the general coupling coefficients defined in Eq. (8).

$$-2\,\underset{1}{\beta}\,\mathcal{R}^{\alpha\beta}{}_{\alpha\beta}+\frac{1}{6}\,\underset{3}{\alpha}\,\mathcal{R}_{\alpha\beta\chi\delta}\,\mathcal{R}^{\alpha\beta\chi\delta}-\frac{2}{3}\,\underset{3}{\alpha}\,\mathcal{R}_{\alpha\chi\beta\delta}\,\mathcal{R}^{\alpha\beta\chi\delta}+\frac{1}{6}\,\underset{3}{\alpha}\,\mathcal{R}^{\alpha\beta\chi\delta}\,\mathcal{R}_{\chi\delta\alpha\beta}+\frac{1}{2}\,\underset{1}{\beta}\,\mathcal{T}_{\alpha\beta\chi}\,\mathcal{T}^{\alpha\beta\chi}+\underset{1}{\beta}\,\mathcal{T}^{\alpha\beta\chi}\,\mathcal{T}_{\beta\alpha\chi}+2\,\underset{1}{\beta}\,\mathcal{T}^{\alpha}{}_{\alpha}{}^{\beta}\,\mathcal{T}^{\chi}{}_{\beta\chi} \quad (12)$$

PSALTer results panel

$$\mathcal{S}==\iiiint\Big(\mathcal{A}^{\alpha\beta\chi}\,\sigma_{\alpha\beta\chi}+f^{\alpha\beta}\,\tau(\Delta+\mathcal{K})_{\alpha\beta}+\underset{1}{\beta}\Big(4\,\partial_{\beta}\mathcal{A}^{\alpha\beta}{}_{\alpha}-4\,\mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}\,\partial_{\beta}f^{\alpha\beta}+4\,\mathcal{A}_{\beta}{}^{\chi}{}_{\chi}\,\partial^{\beta}f^{\alpha}{}_{\alpha}-2\,\partial_{\beta}f^{\chi}{}_{\chi}\,\partial^{\beta}f^{\alpha}{}_{\alpha}-4\,f^{\alpha\beta}\left(\partial_{\beta}\mathcal{A}_{\alpha}{}^{\chi}{}_{\chi}-\partial_{\chi}\mathcal{A}_{\alpha}{}^{\chi}{}_{\beta}\right)-4\,f^{\alpha}{}_{\alpha}\,\partial_{\chi}\mathcal{A}^{\beta\chi}{}_{\beta}-2\,\partial_{\beta}f^{\alpha\beta}\,\partial_{\chi}f_{\alpha}{}^{\chi}+4\,\partial^{\beta}f^{\alpha}{}_{\alpha}\,\partial_{\chi}f_{\beta}{}^{\chi}+4\,\mathcal{A}_{\alpha\chi\beta}\,\partial^{\chi}f^{\alpha\beta}-2\,\partial_{\alpha}f_{\beta\chi}\,\partial^{\chi}f^{\alpha\beta}-\partial_{\alpha}f_{\chi\beta}\,\partial^{\chi}f^{\alpha\beta}+\partial_{\beta}f_{\alpha\chi}\,\partial^{\chi}f^{\alpha\beta}+\partial_{\chi}f_{\alpha\beta}\,\partial^{\chi}f^{\alpha\beta}+\partial_{\chi}f_{\beta\alpha}\,\partial^{\chi}f^{\alpha\beta}\Big)+\frac{1}{3}\,\underset{3}{\alpha}\Big(4\,\partial_{\beta}\mathcal{A}_{\alpha\chi\delta}-2\,\partial_{\beta}\mathcal{A}_{\alpha\delta\chi}+2\,\partial_{\beta}\mathcal{A}_{\chi\delta\alpha}-\partial_{\chi}\mathcal{A}_{\alpha\beta\delta}+\partial_{\delta}\mathcal{A}_{\alpha\beta\chi}-2\,\partial_{\delta}\mathcal{A}_{\alpha\chi\beta}\Big)\partial^{\delta}\mathcal{A}^{\alpha\beta\chi}\Big)[t,x,y,z]\,dz\,dy\,dx\,dt$$

Wave operator

	${}^{0^+}\mathcal{A}^{\parallel}$	${}^{0^+}f^{\parallel}$	${}^{0^+}f^{\perp}$	${}^{0^-}\mathcal{A}^{\parallel}$
${}^{0^+}\mathcal{A}^{\parallel}\,\dagger$	0	0	0	0
${}^{0^+}f^{\parallel}\,\dagger$	0	$-4\,\underset{1}{\beta}\,k^2$	0	0
${}^{0^+}f^{\perp}\,\dagger$	0	0	0	0
${}^{0^-}\mathcal{A}^{\parallel}\,\dagger$	0	0	0	$\underset{3}{\alpha}\,k^2$

	${}^{1^+}\mathcal{A}^{\parallel}{}_{\alpha\beta}$	${}^{1^+}\mathcal{A}^{\perp}{}_{\alpha\beta}$	${}^{1^+}f^{\parallel}{}_{\alpha\beta}$	${}^{1^-}\mathcal{A}^{\parallel}{}_{\alpha}$	${}^{1^-}\mathcal{A}^{\perp}{}_{\alpha}$	${}^{1^-}f^{\parallel}{}_{\alpha}$	${}^{1^-}f^{\perp}{}_{\alpha}$
${}^{1^+}\mathcal{A}^{\parallel}\,\dagger^{\alpha\beta}$	0	0	0	0	0	0	0
${}^{1^+}\mathcal{A}^{\perp}\,\dagger^{\alpha\beta}$	0	0	0	0	0	0	0
${}^{1^+}f^{\parallel}\,\dagger^{\alpha\beta}$	0	0	0	0	0	0	0
${}^{1^-}\mathcal{A}^{\parallel}\,\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}\mathcal{A}^{\perp}\,\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}f^{\parallel}\,\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}f^{\perp}\,\dagger^{\alpha}$	0	0	0	0	0	0	0

	${}^{2^+}\mathcal{A}^{\parallel}{}_{\alpha\beta}$	${}^{2^+}f^{\parallel}{}_{\alpha\beta}$	${}^{2^-}\mathcal{A}^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^+}\mathcal{A}^{\parallel}\,\dagger^{\alpha\beta}$	0	0	0
${}^{2^+}f^{\parallel}\,\dagger^{\alpha\beta}$	0	$2\,\underset{1}{\beta}\,k^2$	0
${}^{2^-}\mathcal{A}^{\parallel}\,\dagger^{\alpha\beta\chi}$	0	0	0

Saturated propagator

	${}^{0^+}\sigma^{\parallel}$	${}^{0^+}\tau^{\parallel}$	${}^{0^+}\tau^{\perp}$	${}^{0^-}\sigma^{\parallel}$
${}^{0^+}\sigma^{\parallel}\dagger$	0	0	0	0
${}^{0^+}\tau^{\parallel}\dagger$	0	$-\frac{1}{4\beta_1 k^2}$	0	0
${}^{0^+}\tau^{\perp}\dagger$	0	0	0	0
${}^{0^-}\sigma^{\parallel}\dagger$	0	0	0	$\frac{1}{\alpha_3 k^2}$

	${}^{1^+}\sigma^{\parallel}{}_{\alpha\beta}$	${}^{1^+}\sigma^{\perp}{}_{\alpha\beta}$	${}^{1^+}\tau^{\parallel}{}_{\alpha\beta}$	${}^{1^-}\sigma^{\parallel}{}_{\alpha}$	${}^{1^-}\sigma^{\perp}{}_{\alpha}$	${}^{1^-}\tau^{\parallel}{}_{\alpha}$	${}^{1^-}\tau^{\perp}{}_{\alpha}$
${}^{1^+}\sigma^{\parallel}\dagger^{\alpha\beta}$	0	0	0	0	0	0	0
${}^{1^+}\sigma^{\perp}\dagger^{\alpha\beta}$	0	0	0	0	0	0	0
${}^{1^+}\tau^{\parallel}\dagger^{\alpha\beta}$	0	0	0	0	0	0	0
${}^{1^-}\sigma^{\parallel}\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}\sigma^{\perp}\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}\tau^{\parallel}\dagger^{\alpha}$	0	0	0	0	0	0	0
${}^{1^-}\tau^{\perp}\dagger^{\alpha}$	0	0	0	0	0	0	0

	${}^{2^+}\sigma^{\parallel}{}_{\alpha\beta}$	${}^{2^+}\tau^{\parallel}{}_{\alpha\beta}$	${}^{2^-}\sigma^{\parallel}{}_{\alpha\beta\chi}$
${}^{2^+}\sigma^{\parallel}\dagger^{\alpha\beta}$	0	0	0
${}^{2^+}\tau^{\parallel}\dagger^{\alpha\beta}$	0	$\frac{1}{2\beta_1 k^2}$	0
${}^{2^-}\sigma^{\parallel}\dagger^{\alpha\beta\chi}$	0	0	0

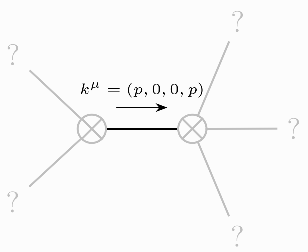
Source constraints

Spin-parity form	Covariant form	Multiplicities
${}^{0^+}\tau^{\perp} == 0$	$\partial_\beta\partial_\alpha\tau\,(\Delta+\mathcal{K})^{\alpha\beta} == 0$	1
${}^{0^+}\sigma^{\parallel} == 0$	$\partial_\beta\sigma^{\alpha}{}_{\alpha}{}^{\beta} == 0$	1
${}^{1^-}\tau^{\perp\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau\,(\Delta+\mathcal{K})^{\alpha\beta}$	3
${}^{1^-}\tau^{\parallel\,\alpha} == 0$	$\partial_\chi\partial_\beta\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\beta\chi} == \partial_\chi\partial^\chi\partial_\beta\tau\,(\Delta+\mathcal{K})^{\beta\alpha}$	3
${}^{1^-}\sigma^{\perp\,\alpha} == 0$	$\partial_\chi\partial_\beta\sigma^{\beta\alpha\chi} == 0$	3
${}^{1^-}\sigma^{\parallel\,\alpha} == 0$	$\partial_\delta\partial^\alpha\sigma^{\chi}{}_{\chi}{}^{\delta} + \partial_\delta\partial^\delta\sigma^{\chi\alpha}{}_{\chi} == \partial_\delta\partial_\chi\sigma^{\chi\alpha\delta}$	3
${}^{1^+}\tau^{\parallel\,\alpha\beta} == 0$	$\partial_\chi\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\beta\chi} + \partial_\chi\partial^\beta\tau\,(\Delta+\mathcal{K})^{\chi\alpha} + \partial_\chi\partial^\chi\tau\,(\Delta+\mathcal{K})^{\alpha\beta} == \partial_\chi\partial^\alpha\tau\,(\Delta+\mathcal{K})^{\chi\beta} + \partial_\chi\partial^\beta\tau\,(\Delta+\mathcal{K})^{\alpha\chi} + \partial_\chi\partial^\chi\tau\,(\Delta+\mathcal{K})^{\beta\alpha}$	3
${}^{1^+}\sigma^{\perp\,\alpha\beta} == 0$	$\partial_\delta\partial_\chi\partial^\alpha\sigma^{\chi\beta\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\chi\alpha\beta} == \partial_\delta\partial_\chi\partial^\beta\sigma^{\chi\alpha\delta}$	3
${}^{1^+}\sigma^{\parallel\,\alpha\beta} == 0$	$\partial_\delta\partial_\chi\partial^\alpha\sigma^{\chi\beta\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\alpha\chi} == \partial_\delta\partial_\chi\partial^\beta\sigma^{\chi\alpha\delta} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi}$	3
${}^{2^-}\sigma^{\parallel\,\alpha\beta\chi} == 0$	$3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\alpha\sigma^{\delta\beta\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\alpha\sigma^{\delta\beta}{}_{\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\alpha\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\chi\alpha\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\beta\sigma^{\delta\alpha\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\beta\alpha\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\delta\alpha\beta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\alpha\beta\chi} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\alpha\sigma^{\delta}{}_{\delta}{}^{\epsilon} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\delta\beta\epsilon} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\delta\alpha}{}_{\delta} ==$ $3\,\partial_\epsilon\partial_\delta\partial^\chi\partial^\beta\sigma^{\delta\alpha\epsilon} + 3\,\partial_\epsilon\partial^\epsilon\partial^\chi\partial^\beta\sigma^{\delta\alpha}{}_{\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\beta\chi\delta} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\chi\beta\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\alpha\sigma^{\delta\beta\chi} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\chi\sigma^{\alpha\beta\delta} + 2\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\beta\alpha\chi} + 4\,\partial_\epsilon\partial^\epsilon\partial_\delta\partial^\delta\sigma^{\chi\alpha\beta} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\beta\sigma^{\delta}{}_{\delta}{}^{\epsilon} + 3\,\eta^{\beta\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial_\delta\sigma^{\delta\alpha\epsilon} + 3\,\eta^{\alpha\chi}\,\partial_\phi\partial^\phi\partial_\epsilon\partial^\epsilon\sigma^{\delta\beta}{}_{\delta}$	5
${}^{2^+}\sigma^{\parallel\,\alpha\beta} == 0$	$3\,\partial_\delta\partial_\chi\partial^\alpha\sigma^{\chi\beta\delta} + 3\,\partial_\delta\partial_\chi\partial^\beta\sigma^{\chi\alpha\delta} + 2\,\eta^{\alpha\beta}\,\partial_\epsilon\partial^\epsilon\partial_\delta\sigma^{\chi}{}_{\chi}{}^{\delta} == 2\,\partial_\delta\partial^\beta\partial^\alpha\sigma^{\chi}{}_{\chi}{}^{\delta} + 3\left(\partial_\delta\partial^\delta\partial_\chi\sigma^{\alpha\beta\chi} + \partial_\delta\partial^\delta\partial_\chi\sigma^{\beta\alpha\chi}\right)$	5
Total expected gauge generators:		33

Massive spectrum

(There are no massive particles)

Massless spectrum



Massless particle

Pole residue:	$\frac{p^2}{\beta P_1} > 0$
Polarisations:	2

Gauge symmetries

(Not yet implemented in PSALTer)

Unitarity conditions

$\beta_1 > 0$

Validity assumptions

(Not yet implemented in PSALTer)

Key observation: Thus we see that only the odd-parity scalar mode is moving without a mass, as claimed.

Key observation: We have now reached the end of the PSALTer calibration script.