Natural Sciences Tripos (physical) and Engineering Tripos

March 23, 2025

Drawing

- Plot the function $f(x)$ from $0 < x < \infty$, where

$$f(x) \equiv \frac{x \log x}{1 + x^2}. \tag{1}$$

Prize

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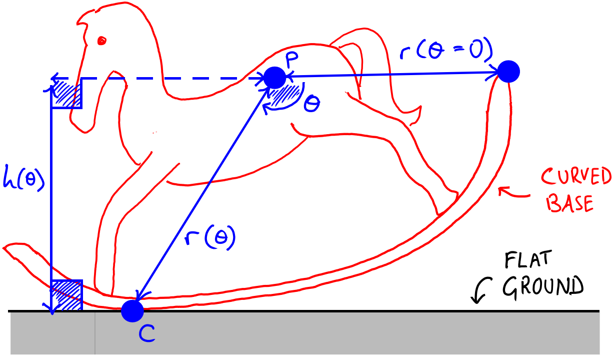
- Now, I'll point to you one box among the two remaining that does **not** contain the prize. Do you keep your choice of the box you selected above, or do you switch?

Volume

- Given that $f(x) \mapsto 1$ as $x \mapsto \infty$, and $f(3/2) \equiv 0$, find I where

$$I \equiv \int_{-\infty}^{\infty} \mathrm{d}x \int_{-\infty}^{\infty} \mathrm{d}y \; f'(x^2 + y^2 + 2x + 5/2). \tag{2}$$

Rocking-horse



- The rocking-horse rolls without slipping on a flat surface. The curved base has point of contact C with the ground and is defined as having a given radius $r(\theta)$ from a fixed point P in the horse, and P is at a height $h(\theta)$ above the ground. Show that

$$h(\theta)^2 = \frac{r^2}{1 + r'(\theta)^2/r^2}. \quad (3)$$

Width

- Consider the following function

$$f(x) \equiv Ce^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \tag{4}$$

for constant $C > 0$, μ and σ, where σ is called the 'width' of $f(x)$.

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- Now consider three functions

$$g(x) \equiv C_1 e^{-\frac{1}{2}(x-4)^2}, \quad h(x) \equiv C_2 e^{-(x-3)^2}, \quad k(x) \equiv g(x)h(x). \tag{5}$$

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- Where is the peak of $k(x)$?
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Population test

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- The test tells us that Alice is a quantum physicist. What is the maximum likelihood that she is?

Grids

- We introduce the concept of a 2×2 'grid' and 'grid multiplication' according to the rules

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \cdot \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \equiv \begin{pmatrix} a_1 b_1 + a_2 b_3 & a_1 b_2 + a_2 b_4 \\ a_3 b_1 + a_4 b_3 & a_3 b_2 + a_4 b_4 \end{pmatrix}. \qquad (6)$$

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$$A \equiv \begin{pmatrix} 1 & 0 \\ -3 & 5 \end{pmatrix} , \quad B \equiv \begin{pmatrix} 2 & 2 \\ 3 & 1 \end{pmatrix} . \tag{7}$$

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- Compute $M \cdot N$ for

$$M \equiv \begin{pmatrix} \cos(\mu) & -\sin(\mu) \\ \sin(\mu) & \cos(\mu) \end{pmatrix} , \quad N \equiv \begin{pmatrix} \cos(\nu) & -\sin(\nu) \\ \sin(\nu) & \cos(\nu) \end{pmatrix} . \tag{8}$$

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- Find μ and ν for

$$\mathsf{N} \equiv \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} , \quad \mathsf{M} \cdot \mathsf{N} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{9}$$

- Plot the curves $f(x, y) = c$, where $c \in \{\pm 1, \pm 1/\sqrt{2}, 0\}$, in the (x, y) plane within the square region $|x| < \pi$ and $|y| < \pi$, where

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- On a **new plot**, do the same for $g(x, y) = 0$ for the new function

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Contours

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- On this **second plot**, add the other curves $g(x,y) = c$, where $c \in \{\pm 1, \pm 1/\sqrt{2}\}$ (*Hint: some of these curves may be points*)

Peeling potatoes

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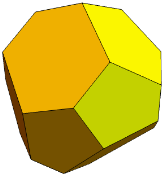
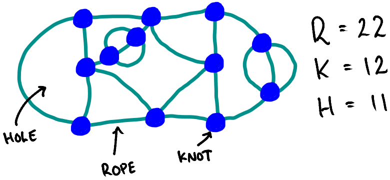
- On another occasion, you discover that it only takes μD seconds to find such potatoes in a *new* pile. This time you *don't* have to peel the potatoes, but you must dice them (i.e. cut them up) into cubes of a given size. It takes ν seconds to dice a $D = \gamma$ potato. What is the best D now?

Fishing nets

- The Euler–Poincaré theorem for *any* convex 3d polyhedron with C corners, E edges and F faces is
$$C - E + F = 2. \quad (12)$$
- A random 2d fishing net is made from R identical ropes joined by K identical knots, such that there are H holes. The *total* underwater weight W (in Newtons) of the net containing one dead fish is
$$W = 4H - 2R. \quad (13)$$
- What are the submerged weights of any rope, any knot, and the fish?





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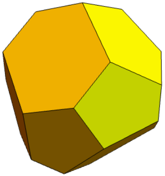

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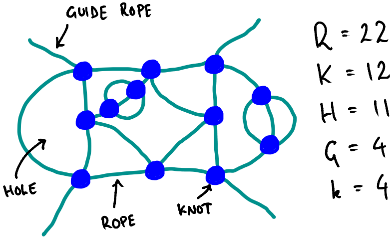
- You just obtained the formula
$$W = 2R - 4K + 4. \qquad (14)$$

Now you discover that each knot connects k ropes. The net is positioned by G guide ropes with free ends, whose weights are neglected. Find k such that for given G, a sufficiently complicated net always sinks with the fish.



Wind turbine

- The wind blows with some velocity v. How does the maximum power output of a wind turbine scale with v?

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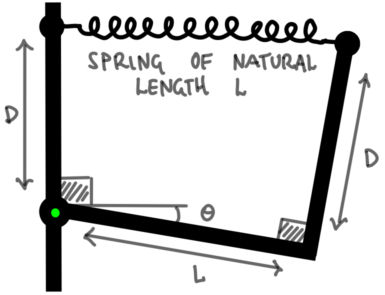
- The pipe is very long. Should I worry about the price of oil or the price of steel?

Drooping beam



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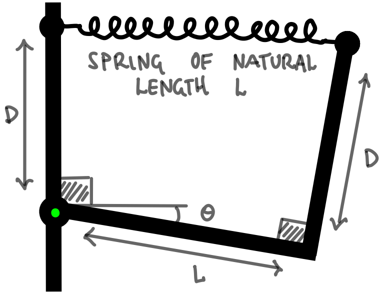

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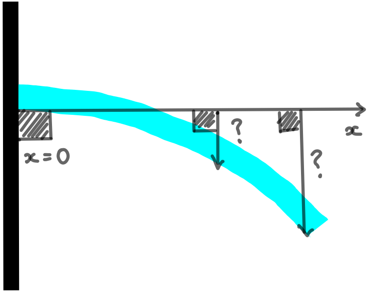
- A long steel beam of uniform cross-section is fixed horizontally to the wall.
- How does the vertical deflection ('droop') of the beam depend on position x along the beam?



Reentry

- The satellite is in a circular orbit around a planet whose atmospheric density decreases exponentially with increasing altitude.
- How does the rate of loss of altitude depend on the altitude?

Combinatorics

- The variables μ, ν, σ and ρ are integers ranging from 0 to 3.
- The so-called '*vector*' A_μ has **four** components A_0, A_1, A_2 and A_3.
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- How many components does the '*tensor*' $F_{\mu\nu} \equiv \frac{1}{2}\left(B_{\mu\nu} + B_{\nu\mu}\right)$ have?
- The '*tensor*' $R_{\mu\nu\sigma\rho}$ has symmetries

$$R_{\mu\nu\sigma\rho} \equiv -R_{\nu\mu\sigma\rho}, \quad R_{\mu\nu\sigma\rho} \equiv -R_{\mu\nu\rho\sigma}, \quad R_{\mu\nu\sigma\rho} \equiv R_{\sigma\rho\mu\nu}. \tag{15}$$

How many components does it have?

Space elevator

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- How does the tension in the cable depend on height above the ground?

- A macroscopic particle of radius r_1 falls through a pipe of radius R.
- What are the chances of collision with a microscopic particle travelling in the opposite direction?

Probability

- A macroscopic particle of radius r_1 falls through a pipe of radius R.
- What are the chances of collision with a microscopic particle travelling in the opposite direction?
- What are the chances of collision with a macroscopic particle of radius r_2 travelling in the opposite direction?

Structures

- A tower must be built from a material whose compressive strain should be equal to 2% everywhere.
- How does the width of the tower change with height?

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- The tower is very tall, reaching far into space. How does the width depend on height now?. (Assume only that gravitational acceleration is $g \equiv g(h)$ without being specific.)

- A light, infinitely flexible pipe of inner radius r conveys water at some velocity v.
- What force must be imposed to bend the pipe by some (small) angle θ?

Fluids

- A light, infinitely flexible pipe of inner radius r conveys water at some velocity v.
- What force must be imposed to bend the pipe by some (small) angle θ?
- What is the force density along a pipe with small bends along its length?.

- Rationalise each of the three quantities

$$\frac{3}{\sqrt{6}} - 9, \quad \frac{1}{2\sqrt{3} - 5\sqrt{2}}, \quad \frac{2\sqrt{2} + 2/\sqrt{2}}{2\sqrt{2} - 2/\sqrt{2}}. \tag{16}$$

- Solve each of the following three equations for finite x

$$e^x = \sqrt{2}, \quad \frac{4 - y^x}{y^{2x} + y^x + 3} = 1, \quad \frac{y^{1/x} + 1}{y^{1/x} - 3} = \frac{2y^{1/x} - 1}{y^{1/x} + 3}. \tag{17}$$

MM1.7

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$$\alpha^4 x^2 = \beta^6 - \gamma^4 y^2, \quad \delta^4 y^2 = \epsilon^6 - \zeta^4 x^2, \tag{20}$$

  depend on the values of α, β, γ, δ, ϵ and ζ?

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- Plot the curve of y in x where

$$y = f(g(x)), \quad f(x) \equiv \sqrt{|x|}, \quad g(x) \equiv |x| - 1. \tag{21}$$

- Compute the value of the sum S, where

$$S = \sum_{N=1}^{M} \sum_{n=1}^{N} n. \tag{22}$$

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- Compute the coefficient of x^5 in the polynomial

$$f(x) \equiv \left(ax^2 + bx + \frac{c}{x} \right)^{10}. \tag{24}$$

- Find an equivalent formula for the difference

$$D = \binom{n-1}{k} - \binom{n-1}{k-1}, \quad n \geq k \geq 1. \tag{25}$$

1. $D = \binom{n}{n-k}\binom{n}{k}$
2. $D = \frac{n+k}{2n}\binom{n-k}{k}$
3. $D = \frac{n-2k}{n}\binom{n}{k}$
4. $D = \frac{n+k}{2n}\binom{n+k}{k}$

MM2.2

- Find an equivalent form of the sum

$$S = \sum_{k=0}^{n} k \binom{n}{k}. \tag{26}$$

1. $S = n2^n$
2. $S = n2^{n-1}$
3. $S = n^{n-1}$
4. $S = 2^n$

- Find an equivalent form of the sum

$$S = \sum_{k=0}^{n} k^2 \binom{n}{k}. \tag{27}$$

1. $S = n^2 2^n$
2. $S = (n - n^2) 2^{2n-1}$
3. $S = n^{n-2}$
4. $S = (n + n^2) 2^{n-2}$

- Find an equivalent form of the sum

$$S = \sum_{j=0}^{k} \binom{m}{j}\binom{n-m}{k-j}, \quad n \geq m \geq 1, \quad n \geq k \geq 1. \tag{28}$$

1. $S = \binom{n-k}{k}$
2. $S = \binom{n+k}{k}$
3. $S = \binom{n}{k}$
4. $S = k\binom{n}{k}$

- How many ways are there to draw **at least** q balls from a bag of $n \geq q$ balls, and then place **exactly** q of the drawn balls in a box?

  1. $\sum_{k=q}^{n} \binom{n}{q}\binom{k}{q}$ or equivalently $2^{n}\binom{n}{n-q}$
  2. $\sum_{k=q}^{n} \binom{n}{k}\binom{k}{q}$ or equivalently $2^{n-q}\binom{n}{q}$
  3. $\sum_{k=0}^{n} \binom{n}{k}\binom{k}{q}$ or equivalently $2^{q}\binom{n}{q}$
  4. $\sum_{k=q}^{n} \binom{n}{k}$ or equivalently $2^{n-q}\binom{n+q}{q}$

MM7.2

- Compute the indefinite integral

$$\int \mathrm{d}x \, \frac{x+3}{x^2+6x+6}. \tag{29}$$

MM2.2

- Let A denote any convex quadrilateral, and let B denote the unique convex quadrilateral whose vertices are at the midpoints of the edges of A. Which of the following statements is true?

  1. B is irregular iff A is irregular.
  2. B is a parallelogram with half the area of A, and the perimeter of B equals the sum of the diagonals of A.
  3. B has one third the area of A, and half the perimeter of A.
  4. B is a square iff A is a rhombus, and the perimeter of B equals the sum of the diagonals of A.

- Consider the following function

$$f(x) \equiv Ce^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \tag{30}$$

for constant $C > 0$, μ and σ, where σ is called the ‘width’ of $f(x)$.

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MM1.7

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- Where is the peak of $f(x)$?
- What does the curve of $f(x)$ look like?
- Now consider three functions

$$g(x) \equiv C_1 e^{-\frac{1}{2}(x-4)^2}, \quad h(x) \equiv C_2 e^{-(x-3)^2}, \quad k(x) \equiv g(x)h(x). \tag{31}$$

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- Where is the peak of $k(x)$?
- What is the 'width' of $k(x)$?

- Plot the curves $f(x, y) = c$, where $c \in \{\pm 1, \pm 1/\sqrt{2}, 0\}$, in the (x, y) plane within the square region $|x| < \pi$ and $|y| < \pi$, where

$$f(x, y) \equiv \cos(x + y). \tag{32}$$

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- On a **new plot**, do the same for $g(x, y) = 0$ for the new function

$$g(x, y) \equiv \cos(x + y)\sin(x - y). \tag{33}$$

M5.10

- Plot the curves $f(x, y) = c$, where $c \in \{\pm 1, \pm 1/\sqrt{2}, 0\}$, in the (x, y) plane within the square region $|x| < \pi$ and $|y| < \pi$, where

$$f(x, y) \equiv \cos(x + y). \tag{32}$$

- On a **new plot**, do the same for $g(x, y) = 0$ for the new function

$$g(x, y) \equiv \cos(x + y)\sin(x - y). \tag{33}$$

- On this **second plot**, add the other curves $g(x, y) = c$, where $c \in \{\pm 1, \pm 1/\sqrt{2}\}$ (*Hint: some of these curves may be points*)

- Cambridge City has 100 000 inhabitants. Ten persons are quantum physicists. We have a test that, with 90% accuracy, determines whether a person is a quantum physicist or not.

M7

- Cambridge City has 100 000 inhabitants. Ten persons are quantum physicists. We have a test that, with 90% accuracy, determines whether a person is a quantum physicist or not.
- The test tells us that Alice is a quantum physicist. What is the maximum likelihood that she is?

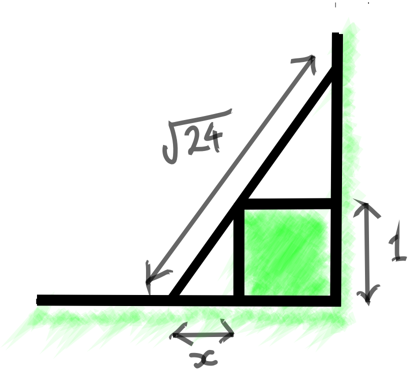
M7

- There are three boxes in front of you and **one** of them contains a prize. You can select any of the boxes. What is the probability that you picked the one with the prize?

M7

- There are three boxes in front of you and **one** of them contains a prize. You can select any of the boxes. What is the probability that you picked the one with the prize?
- Now, I'll point to you one box among the two remaining that does **not** contain the prize. Do you keep your choice of the box you selected above, or do you switch?

Ladder



- The ladder of length $\sqrt{24}$ in some system of units leans against a vertical wall, and makes contact with the corner of a square box of unit side length, which is also touching the wall. What is the distance x between the edge of the box and the foot of the ladder?