A mock

January 16, 2022

Natural Sciences IB: Physics(A)

Instructions

Time 1.5 hours, Answer **ALL QUESTIONS** Calculators are allowed.

Question 1

Explain how the χ^2 test can be used to compare theoretical models with data, including a discussion of "degrees of freedom"

In an experiment, a radioactive source is observed through a varying number, n of sheets of material, and the following counts are measured over one second intervals

number of sheets	1	2	3	4	5	6	7	8
counts in 1s	103	80	79	71	50	52	46	51

The exact nature of the material is unknown, but there are two possibilities, each of which predict a count rate of

$$N_0 e^{-na}$$

where $N_0 = 104.5$ counts s^{-1}, has been determined from a 900 s long integration observation with no sheets of material (i.e. $n = 0$) and a could be either 0.15 or 0.19 depending on the material.

Explain why the value of N_0 is accurately known compared with the counts in 1 s given above

Carry out a χ^2 test to see which value of a is preferred

Plot a suitable graph of the data, including appropriate error bars and the preferred model predictions. Use this to suggest how the model might be improved.

Question 2

A neutral scalar field $\phi = \phi(x, y, t)$ lives on a half-infinite, flat $d = 2$ strip with Cartesian coordinates $0 \leq x < \infty$ and $0 \leq y \leq a$ and evolves over time t with equation of motion

$$\left(\partial_t^2 - \partial_x^2 - \partial_y^2\right) \phi = 0.$$

The Dirichlet condition $\phi(x, 0, t) = \phi(x, a, t) = 0$ is now imposed. Derive the induced mass spectrum for relativistic particles propagating along the strip, as discussed in supervisions

$$m_n \equiv \frac{n\pi}{a}, \quad n \in \mathbb{N}.$$

The system is prepared in the ground state $\phi(x, y, -\infty) = 0$, and at around $t = 0$ an attempt is made to heat the system by imposing a further Dirichlet condition

$$\phi(0, y, t) = \Re\left\{e^{-(iE+\epsilon^2 t)t} f(y)\right\}, \quad E, \epsilon, f(y) \in \mathbb{R}, \quad \epsilon a \ll 1,$$

where $f(y)$ is a very noisy random function (e.g. white noise), and $f(0) = f(a) = 0$. Discuss the condition on E for substantial heating to occur.

At some $x_0 \gg a$ a light particle is first observed at $t_0 \gg a$, followed by a particle of twice the mass at $2t_0$. Infer E in this case and explain why any other particles will be relatively hard to detect.

Question 3

Explain the conditions under which Fresnel diffraction applies

Show that the complex amplitude of the radiation received at an on-axis point a distance z from an annular aperture of radius $r(\ll z)$ and thickness dr illuminated normally by a parallel beam of wavelength λ is given by

$$d\psi \propto K \exp\left(\frac{i\pi r^2}{\lambda z}\right) 2\pi r dr$$

where K is an obliquity factor

Explain with the aid of a phasor diagram how this expression can be used to derive the amplitude received when the annulus is replaced with a circular aperture of radius R. Include in your discussion a definition of Fresnel half-period zones and show that the radius of the n^{th} such zone is given by $\sqrt{n\lambda z}$

A circular aperture 3.8 mm in diameter is illuminated normally with a parallel beam of white light containing radiation over the wavelength range $\lambda = 400$ nm to 700 nm. What wavelengths are missing from the light received on-axis 1 m away from the aperture?